Gauge Invariance and spontaneous symmetry breaking

What happens when the transformation is a gauge invariance rather than a symmetry? Obviously there is no "breaking" — nothing to break! Except:

$$A_\mu \to A_\mu + \partial_\mu \Lambda \qquad \phi \to e^{i\Lambda}\phi$$

If Λ is a constant this doesn't remove any parts of A — this is a true global symmetry and does what global symmetries do: puts particles in ~~their~~ multiplets, enforces charge conservation, etc.

Can this be broken? Sure! The usual potential that causes the field to get a VEV works the usual way. But something interesting happens —

Previously we used the Ward Identity to deduce the existence of a massless particle, the NGB.

$$k_\mu \langle 0|T\{J^\mu(k)\, O_1 \ldots O_n\}|0\rangle = \sum q_i\, e^{ik\cdot x_i} \langle 0|T\{O_1 \ldots O_n\}|0\rangle$$

The same should be true here — but there is a loophole! We discovered that our description of gauge bosons includes fields that create and annihilate modes that are unphysical — they don't couple to physical, gauge invariant observables.

Could it be that the NGB is one of these? Not only "could be" but "alway is"!

Easy way to see this: the current is modified $J^\mu = i D\phi^* \phi + c.c.$

$$D^\mu \phi = (\partial^\mu + ieA^\mu)\phi$$

If we write $\phi \simeq v e^{i\theta}$

$$J^\mu \cong v^2 (\partial^\mu \theta - eA^\mu)$$

We still have to choose a gauge. I will choose Landau Gauge $\partial \cdot A = 0$. Why? $\partial \cdot A$ is a (Lorentz) scalar part of A and can mix with other scalars. The remaining parts of A^μ cannot mix in this way (they will be connected by Lorentz).

Now consider $\partial_\mu J^\mu = 0 \rightarrow \Box\theta = 0$. and θ, the phase in ϕ is our NGB. But θ doesn't couple to A_μ (by gauge choice) and doesn't appear in gauge invariant operators * (This requires some effort to show if the radial, or "Higgs" modes, are included).

If θ is unphysical, why not get rid of it? Perfectly possible: "Unitary Gauge". But this gauge breaks the charge symmetry, so not always a happy situation.

But shouldn't the Gauge field give rise to massless poles? After all, the current is what the gauge field couples to.

No massless poles → the gauge field creates and annihilates MASSIVE spin 1 quanta.

"HIGGS MECHANISM"

(Ex. ~~Abelian~~ Scalar Electrodynamics in R_ξ gauge

Ignoring the radial mode, the Lagrangian may be written

$$e^2 \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (\partial_\mu \theta + A_\mu)^2 \frac{M^2}{2} + \mathcal{L}_\xi$$

where $\mathcal{L}_\xi = -\frac{1}{2\xi}(\partial \cdot A - M^2 \xi \theta)^2$ fixes the gauge

$\xi \to 0$ Landau Gauge. $\xi \to \infty$ is Unitary Gauge

1) Expand the terms and show there is no mixing between A and θ; that is, $\mathcal{L} = \mathcal{L}_\theta + \mathcal{L}_A$

Demonstrate that θ is a free <u>massive</u> particle. What about A?

2) Compute the propagators for θ and A.

3) Verify the Ward Identity.

The Standard Model

All the requisite elements are in place —
we need to

1) specify the gauge degrees of freedom (and the associated global symmetries)

2) Specify the "matter" Fields — all the non-gauge degrees of freedom

3) Any global symmetry constraints

4) All operators of dimension 4 ("relevant")

1) SU(3) color
   SU(2) "weak isospin" or "$SU(2)_L$"
   U(1) hypercharge

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

2) We need to specify the matter fields representations under these groups and Lorentz

Name	SO(3,1)	SU(3)	SU(2)	U(1)
H	(0,0)	1	2	1/2
Q	$(\frac{1}{2},0)$	3	2	1/6
U^c	$(\frac{1}{2},0)$	$\bar{3}$	1	-2/3
D^c	$(\frac{1}{2},0)$	$\bar{3}$	1	+1/3
L	$(\frac{1}{2},0)$	1	2	-1/2
E^c	$(\frac{1}{2},0)$	1	1	1

NB All fermions are $(\frac{1}{2},0)$.

We will NOT impose any additional symmetries

Dimension $[F_{\mu\nu}]=2$ $\qquad [\partial]=1$

$$[\psi_{\text{fermi}}]=3/2$$

$$[H]=1$$

① Only gauge fields

$$\sum_a \left(-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}\right) \equiv -\frac{1}{2}(F,F)$$

Usual Kinetic Term – sum over each generator

$$\tilde{F}^{(a)}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta(a)}$$

$$(F,\tilde{F}) \quad \text{but} \quad (F,\tilde{F}) = \partial_\mu K^\mu$$

total derivatives don't change the EOM $\rightarrow$
No effect classically

Convention: $A_\mu \equiv A^a_\mu T^a \qquad g \equiv e^{i\lambda^a(x)T^a}$

$$A_\mu \to g A_\mu g^{-1} + g i \partial_\mu g^{-1}$$

$$\Phi \to g\,\Phi$$

then $[D\phi] \to g\,D\phi \qquad \mathrm{Tr}\,T^aT^b = \frac{1}{2}\cancel{}\,\delta^{ab}$

and $\quad D_\mu\phi = \partial_\mu\phi - iA_\mu\phi$

$$\cancel{L_{YM}}\; L_{gauge} = -\frac{1}{\cancel{4}2g_i^2}\left(F^{(i)}, F^{(i)}\right)$$

$i = SU(3), SU(2), U(1)$
$\qquad\;\; 3 \qquad\quad 2 \qquad\quad 1 \qquad\qquad g_i$ "gauge coupling"

~~the~~

$$L_{Higgs} = \cancel{}(DH^\dagger, DH) - \frac{\lambda}{2}\left[H^\dagger H - \frac{v^2}{2}\right]^2$$

(NB more derivatives or higher powers are $\delta > 4$)

$$L_\psi = \psi^\dagger \cancel{}\, i\bar{\sigma}^\mu D_\mu \psi$$

"L"

ψ is a large column of all the fermions.

Finally we may form operators of dim 4 involving 2 fermions and H.

the gauge charges tell us there are only 3 types of terms.

$$(Q H)U^c \;;\; (Q H^*)D^c \;;\; (L H^*)E^c$$

these are called "Yukawa" Couplings.

That's all! EXCEPT. Each of fermions is repeicated 3 times → there are 3 families

Gauge Couplings and Masses in EW.

$$\mathcal{A}_\mu = A^a_\mu T^a = A^a_\mu \frac{\sigma^a}{2} \qquad \text{on Higgs Doublet}$$

$$\mathcal{B} = BY = B/2 \qquad \text{'' '' ''}$$

then $\quad D_\mu = \partial_\mu - i\mathcal{A}_\mu - i\mathcal{B}_\mu$

so $\quad D_\mu H^\dagger \cdot D^\mu H = \left| (\mathcal{A}_\mu + \mathcal{B}_\mu) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \right|^2$

$$= \frac{v^2}{2} \frac{(A^1_\mu - iA^2_\mu)(A^1_\mu + iA^2_\mu)}{4} + \frac{v^2}{2} \frac{(A^3_\mu - B_\mu)^2}{4}$$

The gauge kinetic terms are

$$-\frac{1}{4g^2}\left(F^2_{A^1} + F^2_{A^2} + F^2_{A^3}\right) - \frac{1}{4g'^2} F^2_B$$

clearly A^3 and B mix where A^1 and A^2 do not.

A^1 and A^2 are degenerate fields with masses $gv/2$

We combine them into a complex field

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \mp i A^2_\mu)$$

Defining $\quad T^\pm = T^1 \pm i T^2 \qquad$ (No $1/\sqrt{2}$!)

The coupling to $T^\pm$ currents is $g/\sqrt{2}$

The neutral states are more interesting

Clearly one boson stays massless: when $A^3_\mu = B_\mu$. This is the photon, and the current it couples to is called Electromagnetic charge. Since $D_\mu = \partial_\mu - i(A^3_\mu T^3 + B_\mu Y)$

$$= \partial_\mu - i\left[\gamma_\mu Q + Z_\mu (T_3 - \beta Q)\right]$$

when $A^3 = B$ this is Q so

$$\boxed{Q = T^3 + Y}$$

and justifies calling $W^\pm$ "Charged"

What is the electromagnetic coupling?

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} \rightarrow \boxed{e = \frac{gg'}{\sqrt{g^2+g'^2}}}$$

What about the Z coupling?

$$-\frac{1}{4}\left\{\frac{1}{g^2}F^2_{A^3} + \frac{1}{g'^2}F^2_B\right\} = -\frac{1}{4}\left\{\frac{1}{e^2}F^2_\gamma + \frac{1}{x^2}F^2_Z\right\}$$

and $A_3 = \gamma + Z(1-\beta) \quad B = \gamma - \beta Z$

so eliminating the cross term $\boxed{\beta = g'^2/(g^2+g'^2)}$

and then $\boxed{x = \sqrt{g^2+g'^2}}$

and $\boxed{M^2_Z = \sqrt{g^2+g'^2}\, v/2}$

It is conventional to define

$$\cos\theta_W \equiv g/\sqrt{g^2+g'^2} \quad \text{so} \quad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

or $$g = e/\sin\theta_W \qquad g' = \frac{e}{\sin\theta_W}$$

and $$\frac{M_W}{M_Z} = \cos\theta_W$$

Couplings

γ	to	$Q = T^3 + Y$	strength	e
$W^\pm$	to	$T^\pm$	"	$g/\sqrt{2}$
Z	to	$T^3 - \sin^2\theta_W Q$	"	$\sqrt{g^2+g'^2}$

Global Symmetries

Notice we did NOT impose any global symmetries on the model. But nevertheless some symmetries seem to appear:

~~For~~ From our table we can assign

	$U(1)$	$U(1)$
L:	1	0
E^c	-1	0
Q	0	1/3
U^c	0	-1/3
D^c	0	-1/3

The only terms that might violate these are the Yukawas — but they don't!

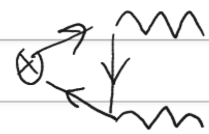
"Lepton Number" and "Baryon Number"

These are called "accidental symmetries" → we can't violate them even if we want too! The constraints of gauge invariance and dim 4, along with our particle content, enforce this. HOWEVER its only true at dim 4 or less

Ex. Construct operators that violate L and B (especially those of low dimension)

* There is a subtlety — although these transformations give rise to conserved currents at the classical level, they are actually violated by EW quantum effects —

Firstly, even at 1-loop, there is an effect called (unfortunately) the "anomaly"

J^μ_B or J^μ_L 

$$\partial_\mu J^\mu_B = \frac{g^2}{32\pi^2} N_{1/2} \, (F, \tilde{F})$$
$$= \partial_\mu J^\mu_L$$

where $F_{\mu\nu}$ includes the SU(2) gauge fields

B+L not conserved! Why doesn't the proton decay?

1) $(F, \tilde{F})$ is a total derivative and shouldn't matter. This isn't quite true, but almost — the effects are strictly non-perturbative $\sim e^{-8\pi^2/g^2}$. This is unobservably small for the weak interactions but is significant for QCD

→ should not ignore $(G, \tilde{G})$ operator

∴ New coupling $\frac{\theta (G, \tilde{G})}{32\pi^2}$ Expt $\bar{\theta} < 10^{-9}$!
"Strong CP Problem"

2) Actually $\Delta B = 3$ so proton is stable anyway.

3) <u>NB</u> B-L conserved "non-anomalous"

But anyway, these accidental symmetries are important - if they were not accidental we would expect operators of dim ≤ 4 to receive contributions from short distances and would no longer have an explanation of the smallness of B and L violation.

$\therefore$ dim ≤ 4 operators are the ones <u>sensitive</u> to short distance physics.

Inadequacies of the SM

1) Doesn't include gravity
$\Longrightarrow$ Gravity is described by a dimensionful parameter, $M_P \sim 10^{19}$ GeV. It is possible to postpone any issue to a high scale * (See later)

2) Doesn't have massive neutrinos.

do last $\rightarrow$ ii) Can always add degrees of freedom
What Q#?

	$SU(3)$	$SU(2)$	$U(1)$
N^c	1	1	1

$\therefore$ Totally neutral. A "singlet neutrino" or often a "sterile neutrino"

Lagrangian? $(LH)N^c$ New Yukawa

And: $M N^c N^c$

Only possible since N^c is a singlet.

This operator violates the global lepton symmetry (but that's OK)

Ex If M very large, we may integrate it out. What does this do?

i)

~~ii~~) No ~~to~~ new light degrees of freedom.

→ Neutrinos are automatically massless aside from higher-d operators

(a "success" ?)

Ex. What is the lowest dim operator for ν mass?

3) Doesn't have a dark matter candidate.

Strong observational evidence (Galactic Rotation Curves; CMB, BAO, LSS, Grav Lensing) implies the existence of non-SM objects behaving today like non-relativistic material.

Remarkably (but not uniquely) new degrees of freedom near the TeV scale fit the bill.

4) Dark Energy

No idea how dark energy fits in

5) Strong CP probem

the operator $\frac{\theta}{32\pi^2} G\tilde{G}$ gives rise to a new coupling constant (* $\bar{\theta} \equiv \theta + \arg\det M$) that violates CP

If we were to set the Yukawas to zero we would have a large global symmetry. We might also ignore the weak gauge couplings

Ex. $Q \times 3$ families $\to SU(3)$
$U^c, D^c \times 3$ families $\to SU(6)$

These couplings break the symmet. in very specific ways (GIM)

The higher dimension (dim 6) operators break the symmetries in different ways, giving rise to flavor effects that don't arise in the SM (except possibly at high loop order)
↳ The scale of these new ops are strongly constrained.

Multi TeV for essentially all of them
Multi 10 TeV for many (most?)
Multi 1000 TeV for some!

7) Hierarchy —

Recall: the physics at high scales contributes to the coefficients of the operators in our effective field theory (called the "SM")

These operator coefficients give us (imprecise) information about high scales

Ex: Absence of proton decay tells us the dim 6 operator has a coefficient less than $\frac{1}{\Lambda^2} \lesssim \left(\frac{1}{10^{16}\,\text{GeV}}\right)^2$

$\therefore$ New physics respects symmetry up to very high scale.

Similar conclusions apply the flavor higher dimension operators.

But this leads to a puzzle —
there are operators in the SM of dim < 4 — these have coefficients that depend sensitively on the short distance theory

There are $\underline{\underline{2}}$ such operators:

① $\mathbb{1}$ the identity. Normally we adjust the vacuum energy to remove this. But in cosmology (or gravity more generally) this operator has consequences and can be measured

$$C_{\mathbb{1}} \sim \Lambda_{CC} \sim \rho_{CC} \sim \pi \, 10^{-11} \, eV^4$$

$$\sim (1 \, TeV)^4 \times \pi \times 10^{-59} \quad !$$

② $H^\dagger H$ $\qquad C_{H^\dagger H} \sim (175 \, GeV)^2$