

DECAY CONSTANTS AND DISTRIBUTION AMPLITUDES OF B MESON IN THE RELATIVISTIC POTENTIAL MODEL

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Under the instructions of my advisor 杨茂志 (Mao-Zhi Yang) 教授

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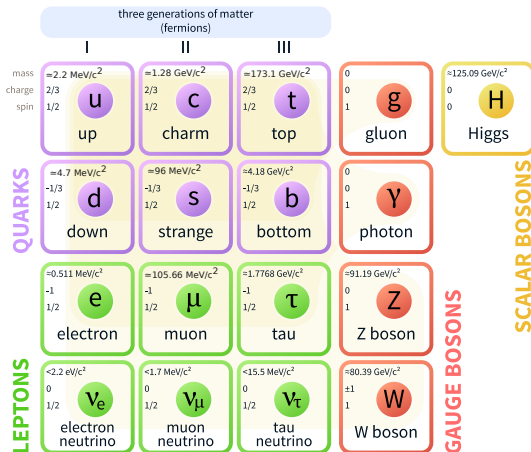
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THE STANDARD MODEL

- SM and NP
 - ATLAS, $t\bar{t}H$ production
 - ν_l, R_K, R_{K^*} , etc
- EW and QCD
 - QED, $\alpha_e \simeq \frac{1}{137.036}$
 - Weak interaction, $G_F \simeq 1.166 \times 10^{-5} \text{GeV}^{-2}$
 - QCD coupling 'constant', $\alpha_s(m_Z) \simeq 0.1181$

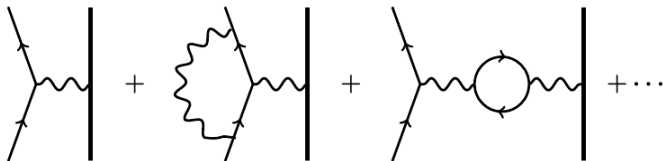
Standard Model of Elementary Particles



By PBS NOVA, Fermilab, Office of Science, U.S. DOE, PDG

PERTURBATION THEORY

- In a perturbatively calculable theory, like QED, many processes can be predicted theoretically order by order to achieve very high accuracy.



- However, due to the running coupling constant $\alpha_s(\mu)$ and quark confinement, analysis of QCD-related processes must take non-perturbative contributions into consideration.

BEAUTY QUARK MODES

- The production, decay channels, and exotic states relevant with **beauty** quark are extensively and accurately measured by high energy experiments, providing us plenty of data used as criteria for the validation of theoretical research.
 - D0, BaBar, Belle, BESIII, LHCb, etc
 - Belle II
- The **wave functions** describing internal interactions and structures of bound states, the **decay constants** and **form factors** presenting key information of hadronization and decay processes require our study into the contributions from both perturbative and non-perturbative regions.

Wave Functions \longrightarrow Mass Spectra \longrightarrow Decay Constants ...

- QCD Sum Rules
- Bethe-Salpeter equation
- Field Correlator Method
- Soft-wall Holographic
- **Potential models**
- Lattice QCD
- ...

RELATIVISTIC POTENTIAL MODEL

Why we choose relativistic potential model for heavy-light mesons:

- concise forms
- relatively simple calculations
- no extra resources
- clear and intuitive physics picture

The wave function from solving the Schrödinger wave equation with relativistic dynamics

$$(H_0 + H')\Psi(\vec{r}) = E\Psi(\vec{r})$$

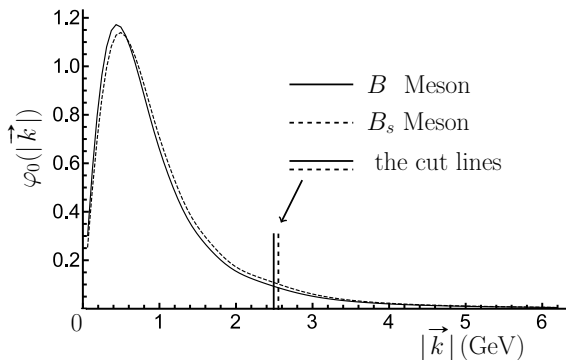
$$H_0 = \sqrt{\vec{k}_1^2 + m_1^2} + \sqrt{\vec{k}_2^2 + m_2^2} + V(r), \quad V(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} + br + c$$

WAVE FUNCTIONS

With conventions for normalization constants, we obtain wave functions for B mesons in momentum space:

$$\int d^3k \left| \Psi_0(\vec{k}) \right|^2 = 1$$

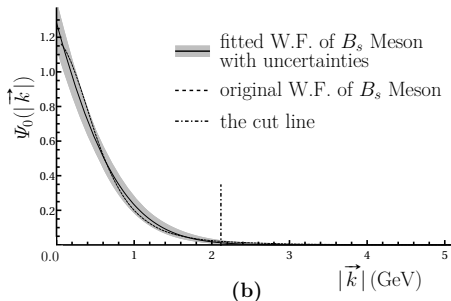
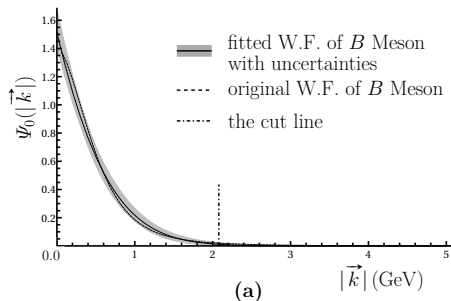
$$\Psi_0(\vec{k}) = \frac{\varphi_0(\vec{k})}{|\vec{k}|} Y_{00}(\theta, \phi)$$



Applying them in meson-mass-spectra calculation, we get results consistent with Expt.

In practice, we make use of the following fitted forms of the wave functions:

$$\psi_0(\vec{k}) = a_1 e^{a_2 |\vec{k}|^2 + a_3 |\vec{k}| + a_4}$$



EXPRESSIONS OF DECAY CONSTANTS

In general, the decay constant of a pseudoscalar meson is defined by the matrix element of the axial current between the meson and the vacuum state:

$$\langle 0 | \bar{q} \gamma^\mu \gamma^5 Q | P \rangle = i f_P P^\mu$$

The pseudoscalar meson as a bound state of a quark and antiquark system can be described by

$$|P(\vec{P})\rangle = \frac{1}{\sqrt{N_L}} \frac{1}{\sqrt{3}} \sum_i \int d^3 k_q d^3 k_Q \delta^{(3)}(\vec{P} - \vec{k}_q - \vec{k}_Q) \Psi_0(\vec{k}_q) \\ \frac{1}{\sqrt{2}} \left[c^{i\dagger}(\vec{k}_Q, \uparrow) b^{i\dagger}(\vec{k}_q, \downarrow) - c^{i\dagger}(\vec{k}_Q, \downarrow) b^{i\dagger}(\vec{k}_q, \uparrow) \right] |0\rangle$$

Holding the four-momentum conservation law, $k_q + k_Q = P$, we adopt the ACCMM scenario,

$$\begin{aligned}E_q + E_Q &= m_P, \\E_q^2 &= m_q^2 + |\vec{k}|^2, \\m_Q^2(\vec{k}) &= E_Q^2 - |\vec{k}|^2.\end{aligned}$$

and we also restrict the heavy quark mass, $m_Q(\vec{k}) \geq 0$, which responds to the **cut lines** in above figures.

The decay constant:

$$f_P = \sqrt{\frac{3}{(2\pi)^3 m_P}} \int d^3k \Psi_0(\vec{k}) \frac{(E_q + m_q)(E_Q + m_Q) - |\vec{k}|^2}{\sqrt{E_q E_Q (E_q + m_q)(E_Q + m_Q)}},$$

NUMERICAL RESULTS

Parameters:

$$m_s = 0.32 \text{ GeV}, \quad m_u = m_d = 0.06 \text{ GeV}, \quad m_b = 4.99 \text{ GeV},$$

and the mesons' masses are taken from PDG

$$m_B = 5.28 \text{ GeV}, \quad m_{B_s} = 5.37 \text{ GeV}.$$

Our prediction of B mesons' decay constants:

$$f_B = 219 \pm 15 \text{ MeV}, \quad f_{B_s} = 266 \pm 19 \text{ MeV}, \quad f_{B_s}/f_B = 1.21 \pm 0.09.$$

Reference	Method	f_B (MeV)	f_{B_s} (MeV)	f_{B_s}/f_B
this work	RPM*	219 ± 15	266 ± 19	1.21 ± 0.09
Colangelo 91	RPM	230 ± 35	245 ± 37	1.07 ± 0.17
Cvetič 04	QM BS [‡]	196 ± 29	216 ± 32	1.10 ± 0.18
Badalian 07	FCM [£]	182 ± 8	216 ± 8	1.19 ± 0.03
Hwang 09	LFQM [§]	204 ± 31	270.0 ± 42.8	1.32 ± 0.08
HPQCD 11	LQCD (2+1) [¶]	—	$225 \pm 3 \pm 3$	—
FNAL/MILC 11	LQCD (2+1)	$196.9 \pm 5.5 \pm 7.0$	$242.0 \pm 5.1 \pm 8.0$	$1.229 \pm 0.013 \pm 0.023$
HPQCD 12	LQCD (2+1)	$191 \pm 1 \pm 8$	$228 \pm 3 \pm 10$	$1.188 \pm 0.012 \pm 0.013$
Narison 12	QCD SR [†]	206 ± 7	234 ± 5	1.14 ± 0.03
Gelhausen 13	QCD SR	207^{+17}_{-9}	242^{+17}_{-12}	$1.17^{+0.03}_{-0.04}$
HPQCD 13	LQCD (2+1+1)	184 ± 4	224 ± 5	1.217 ± 0.008
ETM 13	LQCD (2+1+1)	196 ± 9	235 ± 9	1.201 ± 0.025
Aoki 14	LQCD (2+1)	$218.8 \pm 6.4 \pm 30.8$	$263.5 \pm 4.8 \pm 36.7$	$1.193 \pm 0.020 \pm 0.044$
RBC/UKQCD 14	LQCD (2+1)	$195.6 \pm 6.4 \pm 13.3$	$235.4 \pm 5.2 \pm 11.1$	$1.223 \pm 0.014 \pm 0.070$
Wang 15	QCD SR	194 ± 15	231 ± 16	1.19 ± 0.10

* Relativistic potential model. † QCD sum rules. ‡ Quark model based on Bethe-Salpeter equation.

¶ lattice-QCD with dynamical quark flavors N_f . § Light-front quark model. £ Field correlator method.

BR OF LEPTONIC DECAYS

The branching ratios of purely leptonic decays of B mesons,

$$\mathcal{B}(B^\pm \rightarrow l^\pm \nu) = \frac{G_F^2 m_l^2 m_B}{8\pi} \left(1 - \frac{m_l^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B,$$

In this work, we obtain

$$\begin{aligned}\mathcal{B}(B^+ \rightarrow e^+ \nu_e) &= (1.17 \pm 0.18) \times 10^{-11}, \\ \mathcal{B}(B^+ \rightarrow \mu^+ \nu_\mu) &= (5.01 \pm 0.78) \times 10^{-7}, \\ \mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau) &= (1.41 \pm 0.22) \times 10^{-4},\end{aligned}$$

with CKM parameter from PDG,

$$|V_{ub}| = (4.09 \pm 0.39) \times 10^{-3}.$$

For instance,

Experiment	Tagging	\mathcal{B} (units of 10^{-4})
Belle	Hadronic	$0.72^{+0.27}_{-0.25} \pm 0.11$
Belle	Semileptonic	$1.25 \pm 0.28 \pm 0.27$
<i>BABAR</i>	Hadronic	$1.83^{+0.53}_{-0.49} \pm 0.24$
<i>BABAR</i>	Semileptonic	$1.7 \pm 0.8 \pm 0.2$

TABLE: Experimental results for $\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau)$.

D.A. IN COORDINATE SPACE (1)

Generalizing the current in the definition of the decay constant to nonlocal operators and utilizing Fierz identity, we obtain the matrix element:

$$\begin{aligned}
 \tilde{\Phi}_{\alpha\beta}(z) &\equiv \langle 0 | \bar{q}_\beta(z) [z, 0] Q_\alpha(0) | \bar{B}(P) \rangle \\
 &= \frac{1}{4} \langle 0 | \bar{q}(z) Q(0) | \bar{B} \rangle I_{\alpha\beta} + \frac{1}{4} \langle 0 | \bar{q}(z) \gamma^5 Q(0) | \bar{B} \rangle (\gamma^5)_{\alpha\beta} \\
 &\quad + \frac{1}{8} \langle 0 | \bar{q}(z) \sigma^{\mu\nu} \gamma^5 Q(0) | \bar{B} \rangle (\sigma_{\mu\nu} \gamma^5)_{\alpha\beta} + \frac{1}{4} \langle 0 | \bar{q}(z) \gamma^\mu Q(0) | \bar{B} \rangle (\gamma_\mu)_{\alpha\beta} \\
 &\quad - \frac{1}{4} \langle 0 | \bar{q}(z) \gamma^\mu \gamma^5 Q(0) | \bar{B} \rangle (\gamma_\mu \gamma^5)_{\alpha\beta},
 \end{aligned}$$

where $[z, 0]$ stands for the path-ordered exponential, which is called Wilson line that connects the point 0 and z :

$$[z, 0] \equiv \text{Pexp} \left(i \int_0^z dx^\mu A_\mu(x) \right).$$

D.A. IN COORDINATE SPACE (2)

Using the similar calculations as in decay constants, we obtain:

$$\begin{aligned}
 \langle 0 | \bar{q}(z) Q(0) | \bar{B} \rangle &= 0, \\
 \langle 0 | \bar{q}(z) \gamma^5 Q(0) | \bar{B} \rangle &= -i f_B m_B \tilde{\phi}_P, \\
 \langle 0 | \bar{q}(z) \sigma^{\mu\nu} \gamma^5 Q(0) | \bar{B} \rangle &= -i f_B \tilde{\phi}_T (P^\mu z^\nu - P^\nu z^\mu), \\
 \langle 0 | \bar{q}(z) \gamma^\mu Q(0) | \bar{B} \rangle &= 0, \\
 \langle 0 | \bar{q}(z) \gamma^\mu \gamma^5 Q(0) | \bar{B} \rangle &= f_B (i \tilde{\phi}_{A1} P^\mu - m_B \tilde{\phi}_{A2} z^\mu).
 \end{aligned}$$

Introducing a few conventions, $N_B \equiv \frac{i}{f_B} \sqrt{\frac{3}{(2\pi)^3 m_B}}$, $k_q = (E_q, \vec{k})$, and

$$\begin{aligned}
 A_T(k^1, k^2, k^3) &\equiv \Psi_0(\vec{k}) \frac{E_Q + m_Q + E_q + m_q}{\sqrt{E_q E_Q (E_q + m_q) (E_Q + m_Q)}}, & A_+(k^1, k^2, k^3) &\equiv \Psi_0(\vec{k}) \frac{(E_q + m_q)(E_Q + m_Q) + |\vec{k}|^2}{\sqrt{E_q E_Q (E_q + m_q) (E_Q + m_Q)}}, \\
 A(k^1, k^2, k^3) &\equiv \Psi_0(\vec{k}) \frac{E_Q + m_Q - E_q - m_q}{\sqrt{E_q E_Q (E_q + m_q) (E_Q + m_Q)}}, & A_-(k^1, k^2, k^3) &\equiv \Psi_0(\vec{k}) \frac{(E_q + m_q)(E_Q + m_Q) - |\vec{k}|^2}{\sqrt{E_q E_Q (E_q + m_q) (E_Q + m_Q)}},
 \end{aligned}$$

D.A. IN COORDINATE SPACE (3)

where the four independent D.A.s are

$$\tilde{\phi}_P(z) = N_B \int d^3k [-A_+(k^1, k^2, k^3)] e^{-ik_q \cdot z},$$

$$\tilde{\phi}_T(z) = N_B \int d^3k \left[\frac{1}{3} \sum_i \int_0^{k^i} A_T(\eta, \dots) \eta d\eta \right] e^{-ik_q \cdot z},$$

$$\tilde{\phi}_{A2}(z) = N_B \int d^3k \left[\frac{1}{3} \sum_i \int_0^{k^i} A(\eta, \dots) \eta d\eta \right] e^{-ik_q \cdot z},$$

$$\tilde{\phi}_{A1}(z) = N_B \int d^3k [-A_-(k^1, k^2, k^3) - E_q A(k^1, k^2, k^3)] e^{-ik_q \cdot z}.$$

for the expressions with ellipsis in parenthesis:

$$\sum_i \int_0^{k^i} A_T(\eta, \dots) \eta d\eta = \int_0^{k^1} A_T(\eta, k^2, k^3) \eta d\eta + \int_0^{k^2} A_T(k^1, \eta, k^3) \eta d\eta + \int_0^{k^3} A_T(k^1, k^2, \eta) \eta d\eta.$$

D.A. IN COORDINATE SPACE (4)

Combine these independent D.A.s together, we finally get the D.A. in coordinate space,

$$\tilde{\Phi}_{\alpha\beta}(z) = \frac{-if_B}{4} \left\{ \left[m_B \tilde{\phi}_P + \frac{1}{2} \tilde{\phi}_T (P^\mu z^\nu - P^\nu z^\mu) \sigma_{\mu\nu} + (\tilde{\phi}_{A1} P^\mu + im_B \tilde{\phi}_{A2} z^\mu) \gamma_\mu \right] \gamma^5 \right\}_{\alpha\beta}$$

Considering a decay process which can be expressed as a convolution from

$$F = \int d^4z \tilde{\Phi}_{\alpha\beta}(z) \tilde{T}_{\beta\alpha}(z).$$

Insert proper Fourier transformations,

$$F = \int d^3l \Phi_{\alpha\beta}(l) T_{\beta\alpha}(l) \Big|_{l^2=m_q^2}.$$

D.A. IN MOMENTUM SPACE (1)

Thus, the distribution amplitudes in momentum space are:

$$\Phi_{\alpha\beta}(k) = \left\{ \frac{-if_B m_B}{4} \left[\phi_P(k) + \frac{i}{2} \phi_T(k) \sigma_{\mu\nu} \left(v^\mu \frac{\partial}{\partial k_\nu} - v^\nu \frac{\partial}{\partial k_\mu} \right) + \left(\phi_{A1}(k) \not{v} - \phi_{A2}(k) \gamma_\mu \frac{\partial}{\partial k_\mu} \right) \right] \gamma^5 \right\}_{\alpha\beta}$$

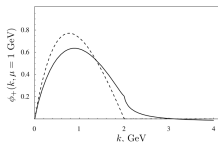
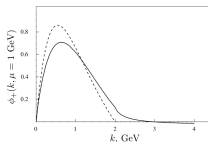
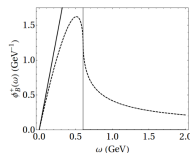
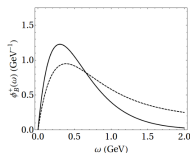
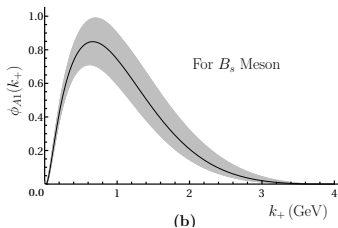
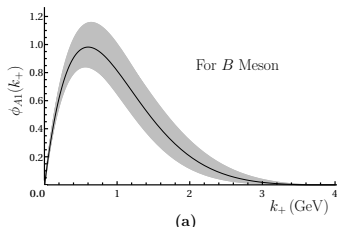
where,

$$\begin{aligned} \phi_P(k_q^\mu) &= -N_B A_+(k^1, k^2, k^3), & \phi_T(k_q^\mu) &= \frac{N_B}{3} \sum_i \int_0^{k^i} A_T(\eta, \dots) \eta \, d\eta \\ \phi_{A1}(k_q^\mu) &= -N_B [A_-(k^1, k^2, k^3) + E_q A(k^1, k^2, k^3)], & \phi_{A2}(k_q^\mu) &= \frac{N_B}{3} \sum_i \int_0^{k^i} A(\eta, \dots) \eta \, d\eta. \end{aligned}$$

For comparison with generally used LCDA ϕ_B^+ ,

$$k_\pm = \frac{E_q \pm k^3}{\sqrt{2}}, \quad k_\perp^\mu = (0, k^1, k^2, 0), \quad \phi_B^+(x) \sim \phi_{A1}(k_+) \quad < \text{heavy quark limit} >$$

D.A. IN MOMENTUM SPACE (2)



D.A. IN MOMENTUM SPACE (3)

For simplicity, we introduce a $K(\vec{k})$ function,

$$K(\vec{k}) \equiv \frac{-2N_B \Psi_0(\vec{k})}{\sqrt{E_q E_Q (E_q + m_q)(E_Q + m_Q)}}$$

then, combining all the independent D.A.s in momentum space together, we obtain a very concise form,

$$\Phi_{\alpha\beta}(k_q^\mu) = \frac{-if_B m_B}{4} K(\vec{k}) \left\{ \begin{pmatrix} (E_Q + m_Q)I_{2 \times 2} \\ \vec{k} \cdot \vec{\sigma} \end{pmatrix} \begin{pmatrix} \vec{k} \cdot \vec{\sigma} & (E_q + m_q)I_{2 \times 2} \end{pmatrix} \right\}_{\alpha\beta}$$

D.A. IN MOMENTUM SPACE (4)

For the convenience of connecting with other approaches, we re-express the above distribution amplitudes,

$$\begin{aligned}
 &= \frac{-if_B m_B}{4} K(\vec{k}) \\
 &\cdot \left\{ (E_Q + m_Q) \frac{1+\psi}{2} \left[\left(\frac{k_+}{\sqrt{2}} + \frac{m_q}{2} \right) \not{v}_+ + \left(\frac{k_-}{\sqrt{2}} + \frac{m_q}{2} \right) \not{v}_- - k_\perp^\mu \gamma_\mu \right] \gamma^5 \right. \\
 &\quad \left. - (E_q + m_q) \frac{1-\psi}{2} \left[\left(\frac{k_+}{\sqrt{2}} - \frac{m_q}{2} \right) \not{v}_+ + \left(\frac{k_-}{\sqrt{2}} - \frac{m_q}{2} \right) \not{v}_- - k_\perp^\mu \gamma_\mu \right] \gamma^5 \right\}_{\alpha\beta}.
 \end{aligned}$$

FACTORIZATION APPROACH

In general, for a physics process, its matrix element F^μ , in factorization approach, can be illustrated as:

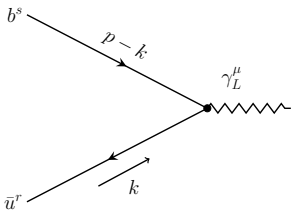
$$F^\mu = \Phi \otimes T$$

Also, we can expand them by the power of α_s in a perturbative way,

$$\begin{aligned} F^\mu &= F^{(0)\mu} + F^{(1)\mu} + \dots = \Phi \otimes T \\ &= [\Phi^{(0)} \otimes T^{(0)}] + [\Phi^{(0)} \otimes T^{(1)} + \Phi^{(1)} \otimes T^{(0)}] + \dots \end{aligned}$$

Therefor, up to one-loop level,

$$\Phi^{(0)} \otimes T^{(1)} = F^{(1)\mu} - \Phi^{(1)} \otimes T^{(0)}$$



$$F_{b\bar{u}}^{(0)\mu} = \langle 0 | \bar{u} \gamma_L^\mu b | \bar{u}^r(k) b^s(p-k) \rangle$$

$$= \frac{1}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0(p-k)^0}} \bar{v}^r(k) \gamma_L^\mu u^s(p-k)$$

$$\Phi_{\alpha\beta}^{(0)b\bar{u}}(\tilde{k}) = \int d^4 z e^{i\tilde{k} \cdot z} \langle 0 | \bar{u}_\beta(z) [z, 0] b_\alpha(0) | \bar{u}^r(k) b^s(p-k) \rangle$$

$$= \frac{1}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0(p-k)^0}} (2\pi)^4 \delta^{(4)}(\tilde{k} - k) \bar{v}_\beta^r(k) u_\alpha^s(p-k)$$

$$F_{b\bar{u}}^{(0)\mu} = \int \frac{d^4 \tilde{k}}{(2\pi)^4} \Phi_{\alpha\beta}^{(0)b\bar{u}}(\tilde{k}) T_{\beta\alpha}^{(0)}(\tilde{k})$$

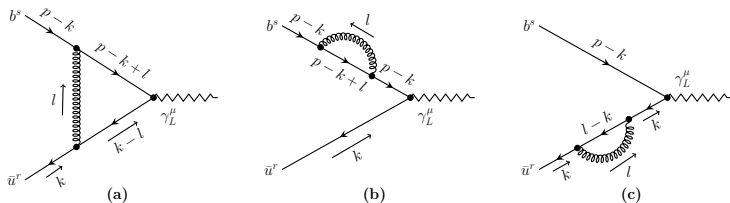
$$= \frac{1}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0(p-k)^0}} \bar{v}_\beta^r(k) T_{\beta\alpha}^{(0)}(k) u_\alpha^s(p-k)$$

And extract the hard-scattering kernel at tree level,

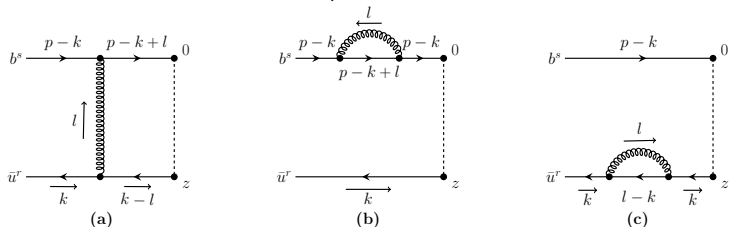
$$T_{\beta\alpha}^{(0)}(k) = (\gamma_L^\mu)_{\beta\alpha}.$$

ONE-LOOP LEVEL (1)

The one-loop Feynman diagrams for matrix element F^μ ,

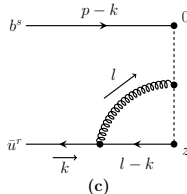
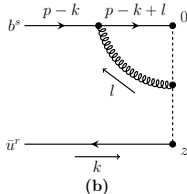
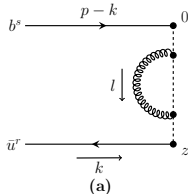


For distribution amplitude $\Phi_{\alpha\beta}$, part one,



ONE-LOOP LEVEL (2)

For distribution amplitude $\Phi_{\alpha\beta}$, part two, related with gluon field in the Wilson line,



$$\begin{aligned}
 F_V^{(1)\mu} &= \Phi_{\alpha\beta}^{(1)V} \otimes T_{\beta\alpha}^{(0)}, \\
 F_{bR}^{(1)\mu} &= \Phi_{\alpha\beta}^{(1)b} \otimes T_{\beta\alpha}^{(0)}, \\
 F_{\bar{u}R}^{(1)\mu} &= \Phi_{\alpha\beta}^{(1)\bar{u}} \otimes T_{\beta\alpha}^{(0)}.
 \end{aligned}$$

Thus,

$$\Phi^{(0)} \otimes T^{(1)} = F^{(1)\mu} - \Phi^{(1)} \otimes T^{(0)} \implies T^{(1)} = 0.$$

- In the relativistic potential model, we obtain wave functions of B mesons via solving the Schrödinger-type wave equation.
- Based on the wave functions, we adopt ACCMM scenario to calculate decay constants of B mesons and branching ratios of the purely leptonic decays thereafter whose results are consistent with experiment data.
- Generalizing to non-local operator current, we further figure out the distribution amplitudes of B mesons including longitudinal and transverse momentum components which should be useful in the study of semileptonic and nonleptonic decays.
- In the end, as an application, we discuss the factorization of the purely leptonic decay up to one-loop level and find that the hard-scattering kernel contribution at one-loop is zero.

- ① Making use of the distribution amplitudes, we shall continue to study the more complicated semileptonic and non-leptonic decay channels.
- ② Carry forward the distribution amplitudes of B mesons to one-loop level and figure out the dominance of next-to-leading order contributions.
- ③ Try to bring the charm mesons into this research scenarios...

THANK YOU

Thank You!