Studys of $D_s \to a_0(980)\ e^+\nu_e$ in the light cone sum rules

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1. introduction

- The inner structure of scalar mesons $a_0(980)$, $f_0(980)$ and $f_0(500)$ has been controversial for over three decades, which make it one of the alluring issues in contemporary particle physics.
- Two possible scenarios are suggested:

  Scenario1: they are the lowest lying two quark bound states

  Scenario2: they are four-quark nonet and $a_0(1450)$, $f_0(1370)$ is treated as the lowest lying two-quark bound state.
- The B decays involving $a_0(980)$ has been explored, but the D_s decays involving $a_0(980)$ haven't been paid much attention.
- In scenario 1, $D_s^+ \to a_0(980)$ can occur through the $D_s^+ \to f_0(980)$ and the isospin breaking $f_0(980) - a_0(980)$ mixing processes.
- Recently, BESIII experiment report the first observation of $f_0(980) - a_0(980)$ mixing.
- In the lowest lying two quark bound state scenario, the branching ratios of $D_s^+ \to a_0^0(980)e^+\nu_e$ are investigated in the light-cone sum rules approach.

ArXiv:1602.05288 Wei Wang

ArXiv:1802.00583 BESIII Collaboration

2. The quark structure of $a_0(980)$, $f_0(980)$, $f_0(500)$

scenario 1:

$a_0(980)$ is the isovector , so

$a_0^0(980)$: $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

$a_0^{\pm}(980)$: $u\bar{d}$ or $d\bar{u}$

$f_0(980)$ and $f_0(500)$ is the isosinglet , so

$f_0(980)$: $\left[\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\right] sin\theta + s\bar{s}cos\theta$

$f_0(500)$: $\left[\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\right] cos\theta - s\bar{s}sin\theta$

The possible regions for the mixing angle θ: $36^o \pm 2^o$ and $148^o \pm 6^o$

scenario 2:

$a_0^0(980)$: $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})s\bar{s}$

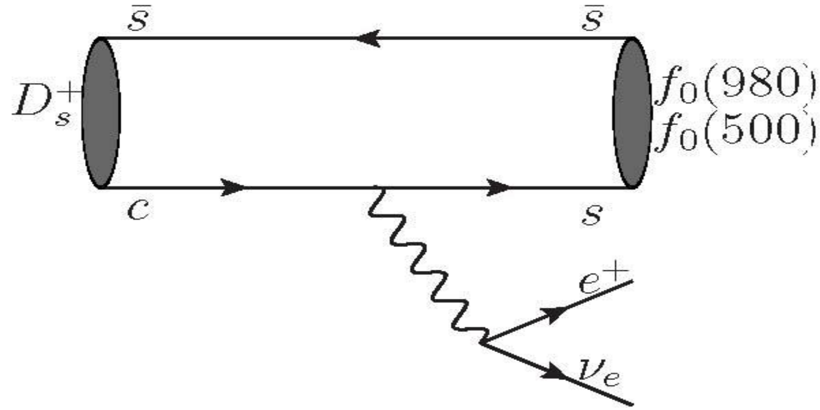
$a_0^{\pm}(980)$: $u\bar{d}s\bar{s}$ or $d\bar{u}s\bar{s}$

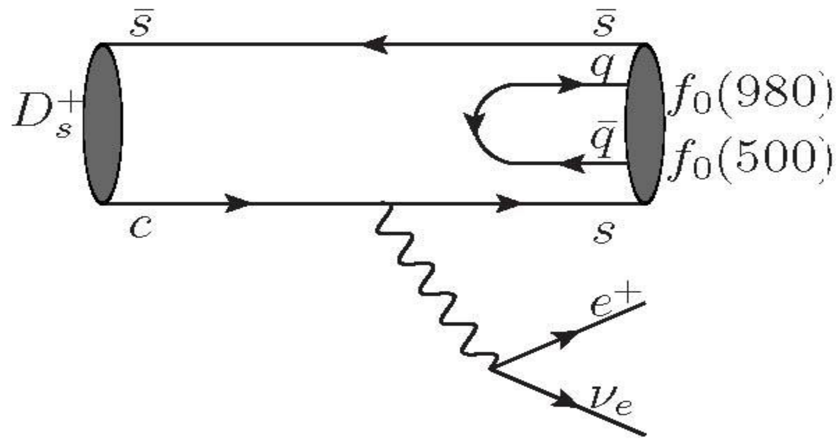
$f_0(980)$: $\left[\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\right] s\bar{s}cos\phi + u\bar{u}d\bar{d}sin\phi$

$f_0(500)$: $-\left[\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\right] s\bar{s}sin\phi + u\bar{u}d\bar{d}cos\phi$

The region for the mixing angle ϕ: $(174.6^{+3.4}_{-3.2})$ or -118.87

Phys.Rew.D 82(2010), 034016 Wei Wang and Cai-Dian lv
Phys.Rew.D 93(2016), 054034 Zhi-Qing Zhang, Si-Yang Wang and Xing-Ke Ma
Phys.Rew.Lett 93(2004), 212002 L. Maiani F. Piccinini A. D. Polosa and V. Riquer
ArXiv:1711.11553 S. S. Agaev K. Azizi and H. Sundu





The values of $Br\left(D_{(s)}^{+} \to f_0(980)(a_0(980), f_0(500))e^+\nu_e\right)$ in scenario 1 and scenario 2 are different, so do $Br(D_s^+ \to f_0(500)e^+\nu_e)/\,Br(D_s^+ \to f_0(980)e^+\nu_e)$.

3. Effective Hamitonian for $c \to s$ transition and parameterizations of the matrix element

$$\mathcal{H}_{eff}(c \to s e^+ \tilde{\nu}_e) = \frac{G_F}{\sqrt{2}} (V_{cs})^+ \bar{s}\gamma^\mu (1-\gamma_5) c \tilde{\nu}_e \gamma_\mu (1-\gamma_5) e$$

$$\langle S(p)|\bar{s}\gamma^\mu c|D_s(p+q)\rangle = 0$$

$$\langle S(p)|\bar{s}\gamma^\mu \gamma_5 c|D_s(p+q)\rangle = -i\left[f_+(q^2)p_\mu + f_-(q^2)q_\mu\right]$$

$$\frac{d\Gamma\left(D_s \to f_0(980) e\nu_e\right)}{dq^2} = \frac{G_F^2 |V_{cs}|^2}{768\pi^3 m_{D_s}^3} \frac{\left(q^2 - m_e^2\right)^2}{q^6} \left\{\left(f_+\left(q^2\right)\right)^2 \left[q^2\left[\left(q^2 + m_{f_0(980)}^2 - m_{D_s}^2\right)^2 - 4q^2 m_{f_0(980)}^2\right]\right.\right.$$

$$\left.+2m_e^2\left[\left(q^2 + m_{f_0(980)}^2 - m_{D_s}^2\right)^2 - q^2 m_{f_0(980)}^2\right]\right] + 6 f_+\left(q^2\right) f_-\left(q^2\right) q^2 m_e^2 \left(m_{D_s}^2 - m_{f_0(980)}^2 - q^2\right)$$

$$\left. 6\left(f_-\left(q^2\right)\right)^2 q^4 m_e^2\right\} \sqrt{\left(m_{D_s}^2 + m_{f_0(980)}^2 - q^2\right)^2 - 4m_{D_s}^2 m_{f_0(980)}^2}$$

arXiv:1011.3901 [hep-ph] Yan-Jun Sun, Zuo-Hong Li and Tao Huang

4. Distribution amplitudes of $f_0(980)$ and $f_0(500)$

The light-cone distributions of S made up of $q_2\bar{q}_1$ can be defined as:

$$\langle S(p)|\bar{q}_2(x)\gamma_\mu q_1(y)|0\rangle = p_\mu \int_0^1 du e^{i(up\cdot x+\bar{u}p\cdot y)}\phi_S(u,\mu),$$

$$\langle S(p)|\bar{q}_2(x)q_1(y)|0\rangle = m_S \int_0^1 du e^{i(up\cdot x+\bar{u}p\cdot y)}\phi_S^s(u,\mu),$$

$$\langle S(p)|\bar{q}_2(x)\sigma_{\mu\nu}q_1(y)|0\rangle = -m_S\left(p_\mu z_\nu - p_\nu z_\mu\right)\int_0^1 du e^{i(up\cdot x+\bar{u}p\cdot y)}\frac{\phi_S^\sigma(u,\mu)}{6},$$

Where $z = x - y, \bar{u} = 1 - u$ and

$$\int_0^1 du\phi_S(u,\mu) = f_S, \qquad \int_0^1 du\phi_S^s(u,\mu) = \int_0^1 du\phi_S^\sigma(u,\mu) = \bar{f}_S$$

The vector current decay constant f_S and the scalar density decay constant $\bar{f}_S$ are defined as:

$$\langle S(p)|\bar{q}_2\gamma_\mu q_1|0\rangle = f_S p_\mu, \qquad \langle S(p)|\bar{q}_2 q_1|0\rangle = m_S \bar{f}_S,$$

$$\phi_S(u,\mu) = \bar{f}_S(\mu)6u\bar{u}\left[B_0(\mu) + \sum_{m=1}^{\infty} B_m(\mu)C_m^{3/2}(2u-1)\right],$$

$$\phi_S^s(u,\mu) = \bar{f}_S(\mu)\left[1 + \sum_{m=1}^{\infty} a_m(\mu)C_m^{1/2}(2u-1)\right],$$

$$\phi_S^\sigma(u,\mu) = \bar{f}_S(\mu)6u\bar{u}\left[1 + \sum_{m=1}^{\infty} b_m(\mu)C_m^{3/2}(2u-1)\right],$$

$$B_0(\mu)=\frac{m_2(\mu)-m_1(\mu)}{m_S}, \qquad B_1(\mu)=\frac{5}{3}\left\langle \xi_\phi^1 \right\rangle,$$

$$B_2(\mu)=\frac{35}{12}\left\langle \xi_\phi^2 \right\rangle-\frac{7}{12}\frac{m_2(\mu)-m_1(\mu)}{m_S}, \qquad B_3(\mu)=\frac{21}{4}\left\langle \xi_\phi^3 \right\rangle-\frac{9}{4}\left\langle \xi_\phi^1 \right\rangle,$$

$$B_4(\mu)=\frac{77}{8}\left\langle \xi_\phi^4 \right\rangle-\frac{77}{12}\left\langle \xi_\phi^2 \right\rangle+\frac{11}{24}\frac{m_2(\mu)-m_1(\mu)}{m_S},$$

$$a_1(\mu)=3\left\langle \xi_s^1 \right\rangle, \qquad a_2(\mu)=\frac{15}{2}\left\langle \xi_s^2 \right\rangle-\frac{5}{2}, \qquad a_3(\mu)=\frac{35}{2}\left\langle \xi_s^3 \right\rangle-\frac{21}{2}\left\langle \xi_s^1 \right\rangle,$$

$$a_4(\mu)=\frac{9}{8}\left(35\left\langle \xi_s^4 \right\rangle-30\left\langle \xi_s^2 \right\rangle+3\right),$$

$$b_1(\mu)=\frac{5}{3}\left\langle \xi_\sigma^1 \right\rangle, \qquad b_2(\mu)=\frac{35}{12}\left\langle \xi_\sigma^2 \right\rangle-\frac{7}{12}, \qquad b_3(\mu)=\frac{21}{4}\left\langle \xi_\sigma^3 \right\rangle-\frac{9}{4}\left\langle \xi_\sigma^1 \right\rangle,$$

$$b_4(\mu)=\frac{77}{8}\left\langle \xi_\sigma^4 \right\rangle-\frac{77}{12}\left\langle \xi_\sigma^2 \right\rangle+\frac{11}{24},$$

as for $f_0(980)$:

$$\left\langle \xi_\phi^1 \right\rangle = -0.47 \pm 0.05, \qquad \left\langle \xi_\phi^2 \right\rangle = 0,$$

$$\left\langle \xi_\phi^3 \right\rangle = -0.20 \pm 0.03, \qquad \left\langle \xi_\phi^4 \right\rangle = 0,$$

as for $f_0(500)$:

$$\left\langle \xi_\phi^1 \right\rangle = -0.22^{+0.04}_{-0.05}, \qquad \left\langle \xi_\phi^2 \right\rangle = 0,$$

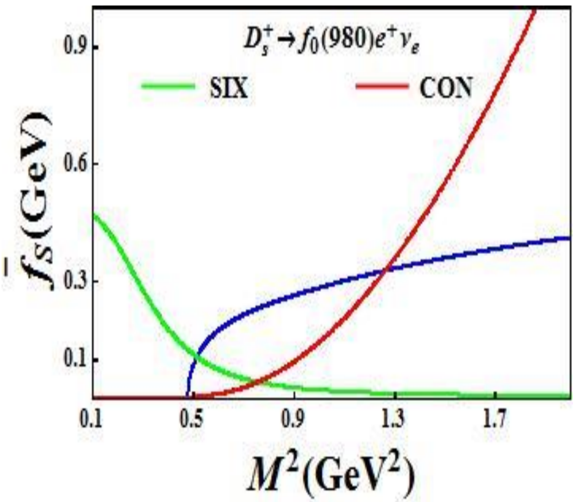
$$\left\langle \xi_\phi^3 \right\rangle = -0.09^{+0.02}_{-0.02}, \qquad \left\langle \xi_\phi^4 \right\rangle = 0,$$

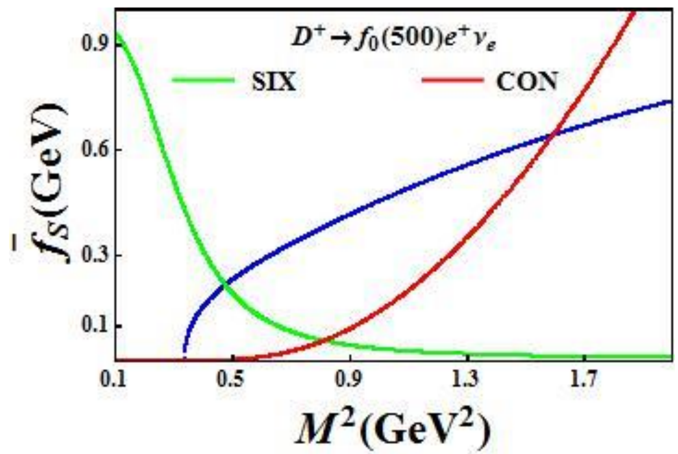
arXiv:1011.3901 [hep-ph] Yan-Jun Sun, Zuo-Hong Li and Tao Huang

arXiv:0508104 [hep-ph] Hai-Yang Cheng, Chun-Khiang Chua and Kwei-Chou Yang

arXiv:0804.2204 [hep-ph] Yu-Ming Wang, M. Jamil Aslam and Cai-Dian lu

- As for the moments $\langle\xi_S^n\rangle$ and $\langle\xi_\sigma^n\rangle$ for twist-3 LCDAs, the sum rules have been computed by hua-Yong Han et al.
- Taking the scalar masses as inputs and setting $n = 0$ for the moment of ϕ_S (such as the moment $\langle\xi_S^{2n}\rangle$), we can derive the sum rules for the decay constant $\bar{f}_S$ of $a_0(980)$, $f_0(980)$ and $f_0(500)$.





- The red line is the ratio of the contribution from the higher excited resonances and continuum states.
- the green line is the ratio of the contribution from the dimension-six condensate.

arXiv:1301.3978 [hep-ph] hua-Yong Han, Xing-Gang Wu, Hai-Bing Fu and Qiong-Lian Zhang we require:

- The contributions from the dimension-six condensate (SIX) hold the fraction less than 5% for $f_0(980)$ and $f_0(500)$ in the total sum rules .

- The contributions from the higher excited resonances and continuum states (CON) hold the fraction less than 20% in the total sum rules.

Subsequently,

- The Borel platform $M^2 \in [0.70,1.10]$ and the decay constant $\bar{f}_S \in [0.21,0.30]$ for $f_0(980)$ are adopted.

- The Borel platform $M^2 \in [0.85,1.10]$ and the decay constant $\bar{f}_S \in [0.39,0.49]$ for $f_0(500)$ are adopted.

- As for $f_0(980)$, the Borel windows for the sum rules of each moment are determined by setting:
  - The contributions from the dimension-six condensate (SIX) hold the fraction less than 5% in the total sum rules.
  - The contributions from the higher excited resonances and continuum states (CON) hold the fraction less than 25% in the total sum rules.
  - the moments mildly varies with respect to the Borel parameter.

the ranges of moments' values and their Borel windows are obtained:

	$\langle\xi_\sigma\rangle$		$\langle\xi_s\rangle$	
	value	$M^2(GeV^2)$	value	$M^2(GeV^2)$
$f_0(980)$	$\langle\xi_\sigma^2\rangle = 0.31 \pm 0.01$	$1.00 \backsim 1.60$	$\langle\xi_s^2\rangle = 0.48$	$1.52 \backsim 1.55$
	$\langle\xi_\sigma^4\rangle = 0.17 \pm 0.01$	$1.51 \backsim 1.70$	$\langle\xi_s^4\rangle = 0.32 \pm 0.01$	$1.24 \backsim 1.59$

	$\langle\xi_\sigma\rangle$		$\langle\xi_s\rangle$	
	value	$M^2(GeV^2)$	value	$M^2(GeV^2)$
$f_0(500)$	$\langle\xi_\sigma^2\rangle = 0.25 \pm 0.04$	$1.00 \backsim 1.60$	$\langle\xi_s^2\rangle = 0.42$	$1.52 \backsim 1.55$
	$\langle\xi_\sigma^4\rangle = 0.16 \pm 0.01$	$1.30 \backsim 1.70$	$\langle\xi_s^4\rangle = 0.27 \pm 0.01$	$1.03 \backsim 1.54$

5. Light cone sum rules for form factors $f_+(q^2)$, $f_-(q^2)$

We consider the following correlators:

$$\Pi_\mu(p,q) = -\int d^4x e^{iqx} \langle S(p) | T\{\bar{q}_2(x)\gamma_\mu\gamma_5 c(x), \bar{c}(0) i\gamma_5 q_1(0)\} | 0 \rangle$$

Inserting the complete set of states between the currents with the same quantum numbers as D_{q_1}:

$$\Pi_\mu(p,q) = i \frac{\langle S(p) | \bar{q}_2(0)\gamma_\mu\gamma_5 c(0) | D_{q_1}(p+q) \rangle \langle D_{q_1}(p+q) | \bar{c}(0) i\gamma_5 q_1(0) | 0 \rangle}{m_{D_{q_1}}^2 - (p+q)^2}$$

$$+ \sum_h i \frac{\langle S(p) | \bar{q}_2(0)\gamma_\mu\gamma_5 c(0) | h(p+q) \rangle \langle h(p+q) | \bar{c}(0) i\gamma_5 q_1(0) | 0 \rangle}{m_h^2 - (p+q)^2},$$

here:

$$\langle D_{q_1}(p+q) | \bar{c}(0) i q_1(0) | 0 \rangle = 0$$

$$\langle D_{q_1}(p+q) | \bar{c}(0) i\gamma_5 q_1(0) | 0 \rangle = \frac{m_{D_{q_1}}^2}{m_c + m_{q_1}} f_{D_{q_1}}$$

$$\langle S(p) | \bar{q}_2(0)\gamma_\mu c(0) | D_{q_1}(p+q) \rangle = 0$$

$$\langle S(p) | \bar{q}_2(0)\gamma_\mu\gamma_5 c(0) | D_{q_1}(p+q) \rangle = -i\left[f_+(q^2) p_\mu + f_-(q^2) q_\mu \right]$$

arXiv:0804.2204 [hep-ph] Yu-Ming Wang M. Jamil Aslam and Cai-Dian Lu

- Making use of the dispersion relation and the quark-hadron duality assumption.

- Performing the Borel transformation, we can arrive at the sum rules for form factors $f_+(q^2)$, $f_-(q^2)$ as

$$f_+(q^2)=\frac{m_c+m_{q_1}}{m_{D_{q_1}}^2 f_{D_{q_1}}}\exp\left(\frac{m_{D_{q_1}}^2}{M^2}\right)\left\{\int_{u_0}^1\frac{du}{u}\exp\left[-\frac{m_c^2+u\bar{u}p^2-\bar{u}q^2}{uM^2}\right]\times\left[-m_c\phi_S(u)+m_S\left(u\phi_S^s(u)+\frac{1}{3}\phi_S^\sigma(u)\right)\right.\right.$$
$$\left.\left.+\frac{m_S}{6uM^2}\phi_S^\sigma(u)\left(m_c^2-u^2p^2+q^2\right)\right]+\frac{m_S}{6}\phi_S^\sigma(u_0)\exp\left(-\frac{s_0^D}{M^2}\right)\frac{m_c^2-u_0^2p^2+q^2}{m_c^2+u_0^2p^2-q^2}\right\},$$

$$f_-(q^2)=\frac{m_c+m_{q_1}}{m_{D_{q_1}}^2 f_{D_{q_1}}}\exp\left(\frac{m_{D_{q_1}}^2}{M^2}\right)\left\{\int_{u_0}^1\frac{du}{u}\exp\left[-\frac{m_c^2+u\bar{u}p^2-\bar{u}q^2}{uM^2}\right]\times\left[m_S\left(\phi_S^s(u)+\frac{1}{6u}\phi_S^\sigma(u)\right)\right.\right.$$
$$\left.\left.-\frac{m_S}{6u^2M^2}\phi_S^\sigma(u)\left(m_c^2+u^2p^2-q^2\right)\right]-\frac{m_S}{6u_0}\phi_S^\sigma(u_0)\exp\left(-\frac{s_0^D}{M^2}\right)\right\},$$

Here,

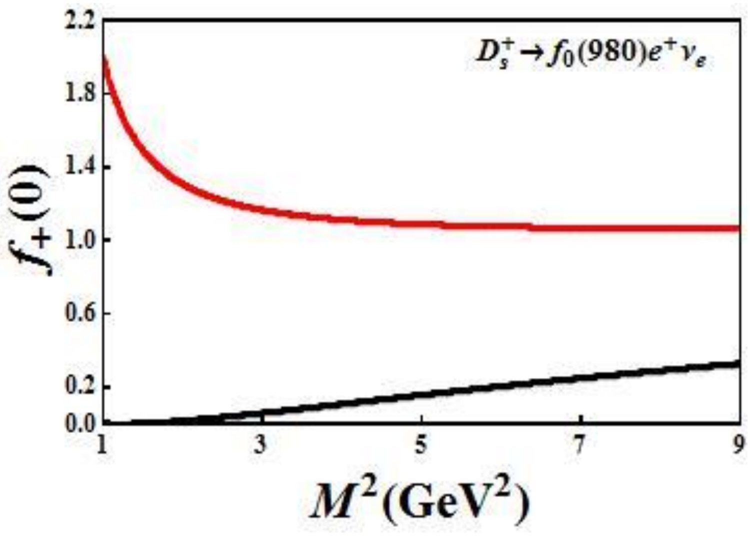
$$u_0 = \frac{\sqrt{\left(s_0^D - q^2 - p^2\right)^2 + 4p^2\left(m_c^2 - q^2\right)} - \left(s_0^D - q^2 - p^2\right)}{2p^2} \qquad \bar{u} = 1 - u$$

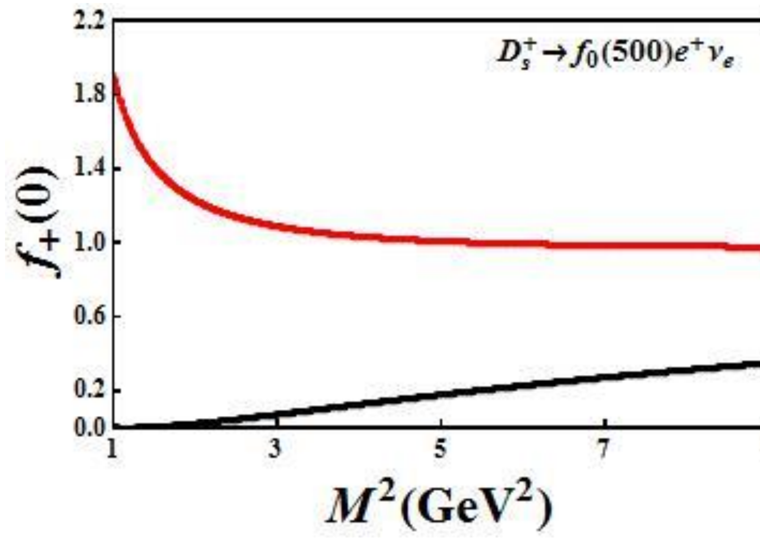
- As for the values of the threshold parameter s_0^D, we adopt $s_0^D = (5.37 \pm 0.01) GeV^2$ corresponding to Ds channel.

- In order to determine the range of Borel parameter M^2, we consider the LCSR for form factors $f_+(0)$, and require that:

  - ✓ The contributions from the higher excited resonances and continuum states hold the fraction less than 20% in the total sum rules.

  - ✓ The value of $f_+(0)$ does not vary drastically within the selected region for the Borel parameter





- The red line denote the values of $f_+(0)$.

- The black line is the ratio of the contribution from the higher Excited resonances and continuum states.

- Subsequently, the Borel platform $M^2 \in [4.54,5.88]$ for $D_s^+ \to f_0(980)e^+\nu_e$ and $M^2 \in [4.43,5.41]$ for $D_s^+ \to f_0(500)e^+\nu_e$ are adopted.

- The parameterized form factor in the double-pole form:

$$f_+(q^2) = \frac{f_+(0)}{1 - a_+ \frac{q^2}{m_{D_{q_1}}^2} + b_+ \frac{q^4}{m_{D_{q_1}}^4}}, \qquad f_-(q^2) = \frac{f_-(0)}{1 - a_- \frac{q^2}{m_{D_{q_1}}^2} + b_- \frac{q^4}{m_{D_{q_1}}^4}}$$

- The numerical results for the parameters $f_i(0)$, a_i and b_i can be fixed by the double-pole fit of the form factors at the small and intermediate q^2 in the LCSR approach.

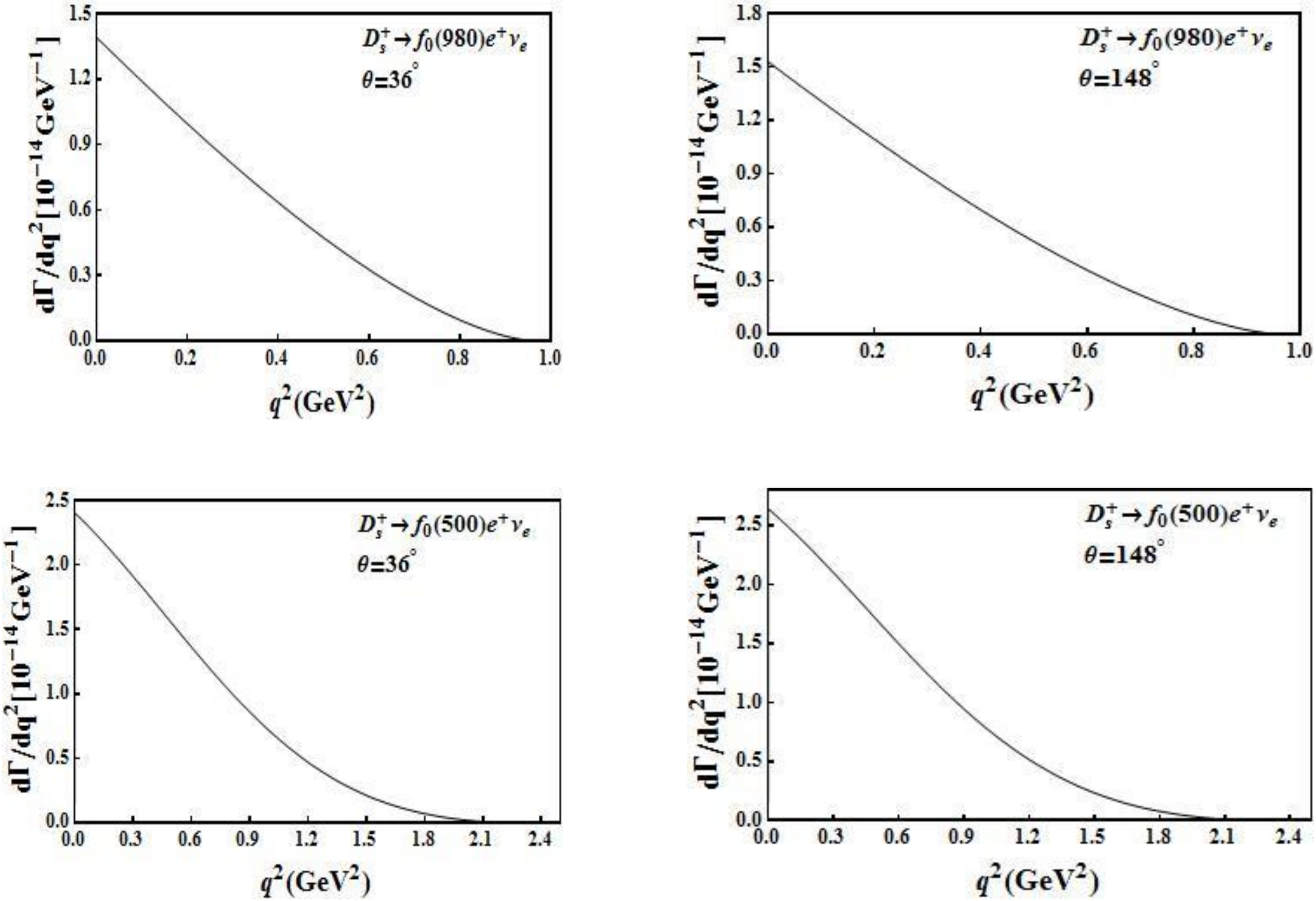
As for $D_s^+ \to f_0(980)e^+\nu_e$:

	$f_+(0)$	a_+	b_+
$f_+(q^2)$	$1.08^{+0.22}_{-0.22}$	$0.52^{+0.09}_{-0.06}$	$0.68^{+0.11}_{-0.13}$
	$f_-(0)$	a_-	b_-
$f_-(q^2)$	$0.30^{+0.07}_{-0.07}$	$1.18^{+0.50}_{-0.33}$	$-0.66^{+1.58}_{-1.00}$

As for $D_s^+ \to f_0(500)e^+\nu_e$:

	$f_+(0)$	a_+	b_+
$f_+(q^2)$	$1.01^{+0.22}_{-0.24}$	$0.63^{+030}_{-0.11}$	$2.77^{+0.93}_{-0.51}$
	$f_-(0)$	a_-	b_-
$f_-(q^2)$	$0.22^{+0.06}_{-0.06}$	$3.69^{+0.37}_{-0.12}$	$4.41^{+1.76}_{-0.70}$

The theoretical uncertainties are caused by varying the Borel parameter M, the decay constant and mass of D meson, the decay constant and the mass of scalar meson, the threshold parameter for scalar meson and the c quark mass, , within their reasonable regions.



$D_s^+ \to f_0(980)e^+\nu_e$
θ=36°
$D_s^+ \to f_0(980)e^+\nu_e$
θ=148°
$D_s^+ \to f_0(500)e^+\nu_e$
θ=36°
$D_s^+ \to f_0(500)e^+\nu_e$
θ=148°
$d\Gamma/dq^2[10^{-14}\mathrm{GeV}^{-1}]$
$q^2(\mathrm{GeV}^2)$

The branching ratio of $D_s^+ \to f_0(980)e^+\nu_e$ at $\theta = (36 \pm 2)^\circ$:

$$Br(D_s^+ \to f_0(980)e^+\nu_e) = \left(4.11^{+1.80}_{-1.51}\right) \times 10^{-3}$$

The branching ratio of $D_s^+ \to f_0(980)e^+\nu_e$ at $\theta = (148 \pm 6)^\circ$:

$$Br(D_s^+ \to f_0(980)e^+\nu_e) = \left(4.52^{+2.05}_{-1.76}\right) \times 10^{-3}$$

The branching ratio of $D_s^+ \to f_0(980)e^+\nu_e$ was measured as:

$$Br(D_s^+ \to f_0(980)e^+\nu_e \to \pi^+\pi^-e^+\nu_e) = (1.30 \pm 0.41) \times 10^{-3}$$

CELO –c Phys. Rev. D 80(2009) 052007

$$Br(D_s^+ \to f_0(980)e^+\nu_e \to \pi^+\pi^-e^+\nu_e) = (2.00 \pm 0.32) \times 10^{-3}$$

CELO Phys. Rev. D 80(2009) 052009

$$Br(D_s^+ \to f_0(980)e^+\nu_e \to \pi^+\pi^-e^+\nu_e) = (1.30 \pm 0.22) \times 10^{-3}$$

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$$Br(f_0(980) \to \pi^+\pi^-) = (0.50^{+0.07}_{-0.09})$$

- The theoretical uncertainties are caused by varying the Borel parameter M, the decay constant and the mass of scalar meson, the threshold parameter for scalar meson and the c quark mass, within their reasonable regions.

arXiv:1303.4403 [hep-ph] Hai-Yang Cheng, Chun-Khiang Chua, Kwei-Chou Yang, Zhi-Qing Zhang

- The measured result of $f_0(980)-a_0^0(980)$ mixing at BESIIII:

$$\xi_{fa}=\frac{Br(J/\psi\to\phi f_0(980)\to\phi a_0^0(980)\to\phi\eta\pi)}{Br(J/\psi\to\phi f_0(980)\to\phi\pi^+\pi^-)}=\begin{cases}(0.99\pm0.35)\times10^{-2}\\(0.41\pm0.25)\times10^{-2}\end{cases}$$

combining with
$$\xi_{fa}=\frac{Br(D_s^+\to a_0^0(980)e^+\nu_e\to\eta\pi e^+\nu_e)}{Br(D_s^+\to f_0(980)e^+\nu_e\to\pi^+\pi^-e^+\nu_e)}$$
we can obtain

$$Br(D_s^+\to a_0^0(980)e^+\nu_e\to\eta\pi e^+\nu_e)=\xi_{fa}\cdot Br(D_s^+\to f_0(980)e^+\nu_e)\cdot Br(f_0(980)\to\pi^+\pi^-)$$

- The numerical results:
  - $\theta=(36\pm2)^\circ$:

$$Br(D_s^+\to a_0^0(980)e^+\nu_e\to\eta\pi e^+\nu_e)=\begin{cases}(2.04^{+1.18}_{-1.10})\times10^{-5}, & \xi_{fa}=(0.99\pm0.35)\times10^{-2}\\(8.43^{+6.45}_{-6.19})\times10^{-6}, & \xi_{fa}=(0.41\pm025)\times10^{-2}\end{cases}$$

  - $\theta=(148\pm6)^\circ$:

$$Br(D_s^+\to a_0^0(980)e^+\nu_e\to\eta\pi e^+\nu_e)=\begin{cases}(2.24^{+1.32}_{-1.24})\times10^{-5}, & \xi_{fa}=(0.99\pm0.35)\times10^{-2}\\(9.27^{+7.16}_{-6.91})\times10^{-6}, & \xi_{fa}=(0.41\pm025)\times10^{-2}\end{cases}$$

- The branching ratio of $Br(D_s^+\to a_0^0(980)e^+\nu_e\to\eta\pi e^+\nu_e)$ can reach 10^{-5} order and a measurement on it will shed light on understanding the nature of $a_0^0(980)$.
- The theoretical uncertainties are large and caused by the decay constant of scalar meson and the mixing intensity ξ_{fa}.

The branching ratio of $D_s^+ \to f_0(500)e^+\nu_e$ at $\theta = (36 \pm 2)^\circ$:

$$Br(D_s^+ \to f_0(500)e^+\nu_e) = \left(7.30^{+3.74}_{-3.15}\right) \times 10^{-3}$$

The branching ratio of $D_s^+ \to f_0(500)e^+\nu_e$ at $\theta = (148 \pm 6)^\circ$:

$$Br(D_s^+ \to f_0(500)e^+\nu_e) = \left(5.93^{+3.53}_{-3.24}\right) \times 10^{-3}$$

The ratio of the branching ratios of $D_s^+ \to f_0(500)e^+\nu_e$ and $D_s^+ \to f_0(980)e^+\nu_e$ in scenario 1:

$$\theta = (36 \pm 2)^\circ: \quad \frac{Br(D_s^+ \to f_0(500)e^+\nu_e)}{Br(D_s^+ \to f_0(980)e^+\nu_e)} = 1.77^{+1.08}_{-1.25}$$

$$\theta = (148 \pm 6)^\circ: \quad \frac{Br(D_s^+ \to f_0(500)e^+\nu_e)}{Br(D_s^+ \to f_0(980)e^+\nu_e)} = 1.31^{+1.24}_{-1.17}$$

7. Background analysis

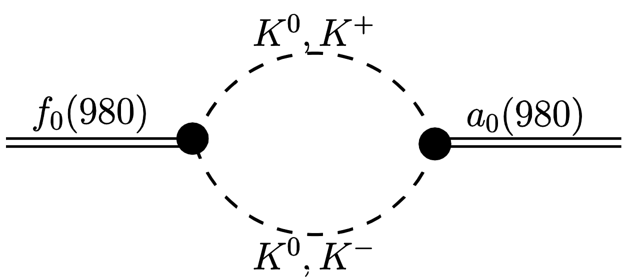
- The signals are obtained with the following decays:

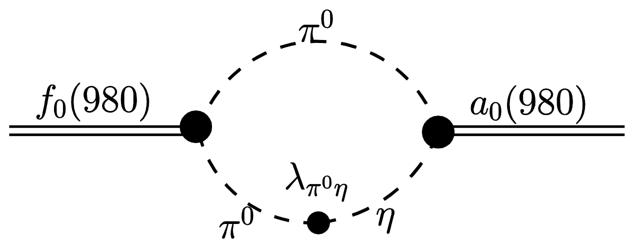
$$D_s^+ \to f_0(980)e^+\nu_e \to a_0^0(980)e^+\nu_e \to \eta\pi e^+\nu_e$$

- $f_0(980) - a_0^0(980)$ mixing is dominated by the difference of the charged and neutral kaon thresholds , so the measured width of the $a_0^0(980)$ is much narrower than the world average.

$I = 0$: $\frac{1}{\sqrt{2}}(K^+K^- + K^0\overline{K}^0)$ $\pi\pi$

$I = 1$: $\frac{1}{\sqrt{2}}(K^+K^- - K^0\overline{K}^0)$ $\eta\pi$





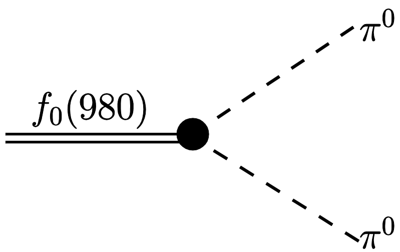
Phys. Lett. 88B (1979) 367-371 N. N. Achasov, S. A. Devyanin and G. N. Shestakov

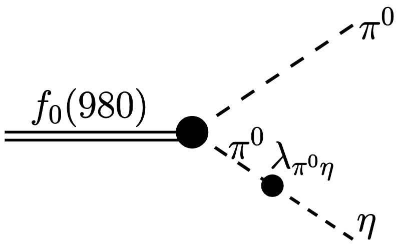
JETP Lett. 72 (2000) 410-414 A. E. Kudryavtsev and V. E. Tarasov

Phys. Rev. D76 (2007) 074028 C. Hanhart, B. Kubis and J. R. Pelaez

- A possible background is:

$$D_s^+ \to f_0(980)e^+\nu_e \to \pi^0\pi^0 e^+\nu_e \to \eta\pi^0 e^+\nu_e$$





$$\Gamma(f_0(980) \to \pi^0\pi^0) \propto \left|g_{f\pi^0\pi^0}\right|^2 \frac{\lambda^{\frac{1}{2}}(m_{f_0(980)}^2, m_{\pi^0}^2, m_{\pi^0}^2)}{16\pi m_{f_0(980)}^3}$$

$$\Gamma(f_0(980) \to \eta\pi^0) \propto 4\left|g_{f\pi^0\pi^0}\right|^2 \frac{\left|\lambda_{\pi^0\eta}\right|^2}{\left(m_\eta^2 - m_{\pi^0}^2\right)^2} \frac{\lambda^{\frac{1}{2}}(m_{f_0(980)}^2, m_\eta^2, m_{\pi^0}^2)}{16\pi m_{f_0(980)}^3}$$

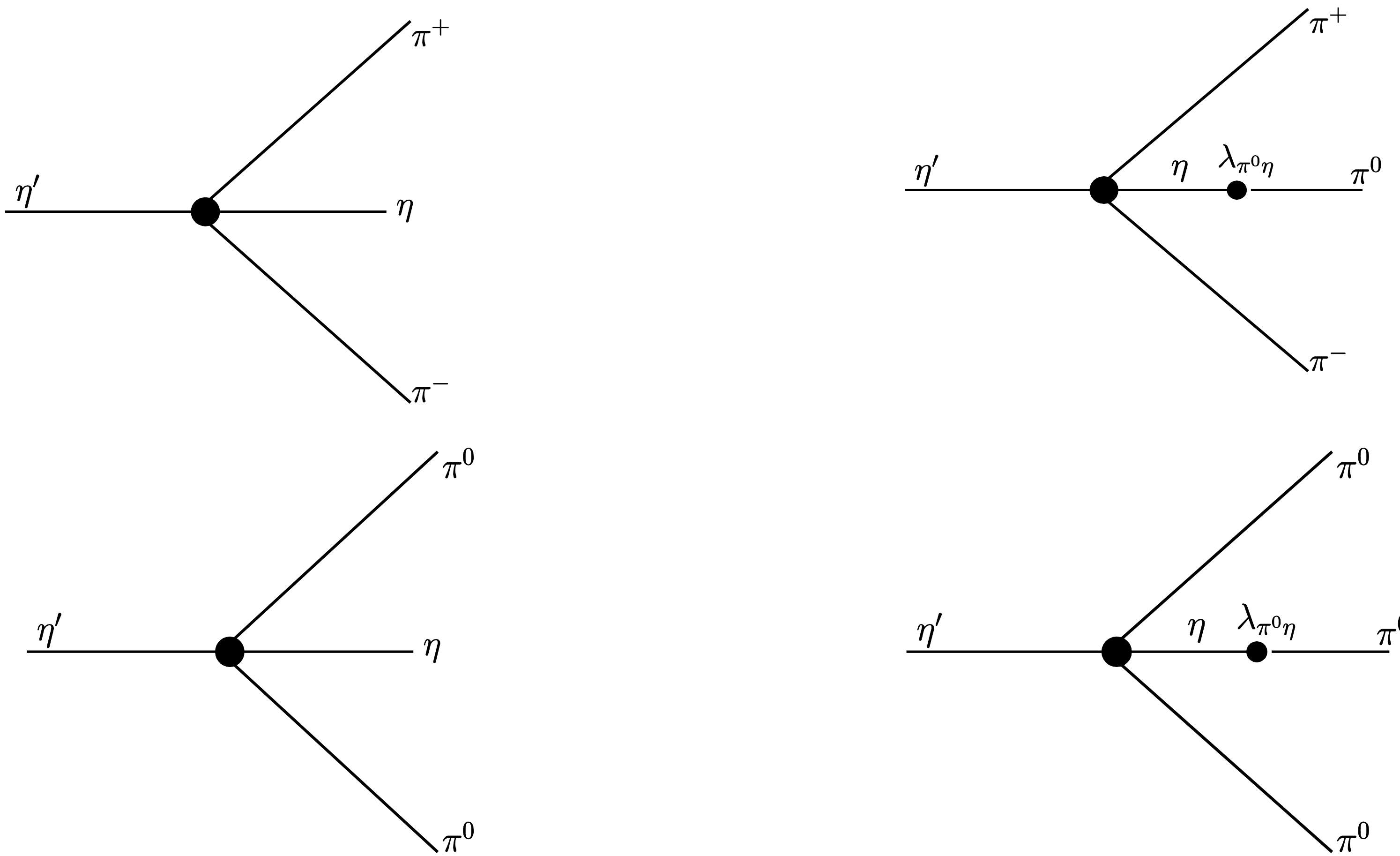
here:

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

so

$$\frac{Br(f_0(980) \to \eta\pi^0)}{Br(f_0(980) \to \pi^0\pi^0)} = 4\frac{\left|\lambda_{\pi^0\eta}\right|^2}{\left(m_\eta^2 - m_{\pi^0}^2\right)^2}\frac{\lambda^{\frac{1}{2}}\left(m_{f_0(980)}^2, m_\eta^2, m_{\pi^0}^2\right)}{\lambda^{\frac{1}{2}}\left(m_{f_0(980)}^2, m_{\pi^0}^2, m_{\pi^0}^2\right)}$$

Here, the $\lambda_{\pi^0\eta}$ vertex corresponding to the $\pi^0 - \eta$ transition is extracted from the $\eta' \to 3\pi$ reaction.



$$\Gamma(\eta' \to \pi^+\pi^-\eta) \propto \left|g_{\eta'\pi^+\pi^-\eta}\right|^2 PS_{\eta'\to\pi^+\pi^-\eta}$$

Here, $PS_{\eta'\to\pi^+\pi^-\eta}$ is the phase space factor for $\eta' \to \pi^+\pi^-\eta$.

$$\Gamma(\eta' \to \pi^+\pi^-\pi^0) \propto \left|g_{\eta'\pi^+\pi^-\eta}\right|^2 \frac{\left|\lambda_{\pi^0\eta}\right|^2}{\left(m_\eta^2 - m_{\pi^0}^2\right)^2} PS_{\eta'\to\pi^+\pi^-\pi^0}$$

so

$$\frac{Br(\eta' \to \pi^+\pi^-\pi^0)}{Br(\eta' \to \pi^+\pi^-\eta)} = \frac{\left|\lambda_{\pi^0\eta}\right|^2}{\left(m_\eta^2 - m_{\pi^0}^2\right)^2} \frac{PS_{\eta'\to\pi^+\pi^-\pi^0}}{PS_{\eta'\to\pi^+\pi^-\eta}}$$

We can obtain

$$\frac{Br(f_0(980) \to \pi^0\eta)}{Br(f_0(980) \to \pi^0\pi^0)} = 4\frac{Br(\eta' \to \pi^+\pi^-\pi^0)}{Br(\eta' \to \pi^+\pi^-\eta)} \frac{PS_{\eta'\to\pi^+\pi^-\eta}}{PS_{\eta'\to\pi^+\pi^-\pi^0}} \frac{\lambda^{\frac{1}{2}}\left(m_{f_0(980)}^2, m_\eta^2, m_{\pi^0}^2\right)}{\lambda^{\frac{1}{2}}\left(m_{f_0(980)}^2, m_{\pi^0}^2, m_{\pi^0}^2\right)}$$

As for $\eta' \to \pi^0\pi^0\pi^0$, we can obtain

$$\frac{Br(f_0(980) \to \pi^0\eta)}{Br(f_0(980) \to \pi^0\pi^0)} = 4\frac{Br(\eta' \to \pi^0\pi^0\pi^0)}{Br(\eta' \to \pi^0\pi^0\eta)} \frac{PS_{\eta'\to\pi^0\pi^0\eta}}{PS_{\eta'\to\pi^0\pi^0\pi^0}} \frac{\lambda^{\frac{1}{2}}\left(m_{f_0(980)}^2, m_\eta^2, m_{\pi^0}^2\right)}{\lambda^{\frac{1}{2}}\left(m_{f_0(980)}^2, m_{\pi^0}^2, m_{\pi^0}^2\right)}$$

Phys. Rev. 175 (1968) 2054-2057 Brian G. Kenny

Nuclear Physics B20 (1970) 23-44 H. Osborn and D. J. Wallace

Phys. Rev. D 34 (1986) 2784-2789 S. A. Coon, B. H. J. McKellar and M. D. Scadron

$$Br(f_0(980) \to \pi^0\pi^0) = \frac{Br(f_0(980) \to \pi^+\pi^+)}{2} = \left(0.25^{+0.04}_{-0.05}\right)$$

$$\frac{Br(\eta' \to \pi^+\pi^-\pi^0)}{Br(\eta' \to \pi^+\pi^-\eta)} = (8.8 \pm 1.2) \times 10^{-3}$$

$$\frac{PS_{\eta'\to\pi^+\pi^-\eta}}{PS_{\eta'\to\pi^+\pi^-\pi^0}} = 0.059$$

$$\frac{\lambda^{\frac{1}{2}}\left(m^2_{f_0(980)}, m^2_{\eta}, m^2_{\pi^0}\right)}{\lambda^{\frac{1}{2}}\left(m^2_{f_0(980)}, m^2_{\pi^0}, m^2_{\pi^0}\right)} = 0.684$$

$$\frac{Br(\eta' \to \pi^0\pi^0\pi^0)}{Br(\eta' \to \pi^0\pi^0\eta)} = (16.9 \pm 1.4) \times 10^{-3}$$

$$\frac{PS_{\eta'\to\pi^0\pi^0\eta}}{PS_{\eta'\to\pi^0\pi^0\pi^0}} = 0.065$$

arXiv:1303.4403 [hep-ph] Hai-Yang Cheng, Chun-Khiang Chua, Kwei-Chou Yang, Zhi-Qing Zhang

Chin. Phys. C42 (2018) no.4, 042002, Shuang-shi Fang, Andrzej Kupsc and Dai-hui Wei

- $\theta = (36 \pm 2)^{\circ}$:

$$Br(D_s^+ \to f_0(980)e^+\nu_e \to \eta\pi e^+\nu_e) = \begin{cases} (1.46^{+0.71}_{-0.64}) \times 10^{-6}\ , & \eta' \to \pi^+\pi^-\eta \\ (3.09^{+1.46}_{-1.31}) \times 10^{-6}, & \eta' \to \pi^0\pi^0\eta \end{cases}$$

$$Br(D_s^+ \to a_0^0(980)e^+\nu_e \to \eta\pi e^+\nu_e) = \begin{cases} (2.04^{+1.18}_{-1.10}) \times 10^{-5}, & \xi_{fa} = (0.99 \pm 0.35) \times 10^{-2} \\ (8.43^{+6.45}_{-6.19}) \times 10^{-6}, & \xi_{fa} = (0.41 \pm 025) \times 10^{-2} \end{cases}$$

- $\theta = (148 \pm 6)^{\circ}$:

$$Br(D_s^+ \to f_0(980)e^+\nu_e \to \eta\pi e^+\nu_e) = \begin{cases} (1.61^{+0.80}_{-0.74}) \times 10^{-6}, & \eta' \to \pi^+\pi^-\eta \\ (3.40^{+1.66}_{-1.51}) \times 10^{-6}, & \eta' \to \pi^0\pi^0\eta \end{cases}$$

$$Br(D_s^+ \to a_0^0(980)e^+\nu_e \to \eta\pi e^+\nu_e) = \begin{cases} (2.24^{+1.32}_{-1.24}) \times 10^{-5}, & \xi_{fa} = (0.99 \pm 0.35) \times 10^{-2} \\ (9.27^{+7.16}_{-6.91}) \times 10^{-6}, & \xi_{fa} = (0.41 \pm 025) \times 10^{-2} \end{cases}$$

- Phenomenological determination the branching ratio of $f_0(980) \to \pi^0\eta$

The quark-flavor basis:

$$\eta_u = u\bar{u} \qquad \eta_d = d\bar{d} \qquad \eta_s = s\bar{s}$$

The isoscalar and isovector basis:

$$\eta_+ = \frac{1}{\sqrt{2}}\left[u\bar{u} + d\bar{d}\right] \qquad \eta_- = \frac{1}{\sqrt{2}}\left[u\bar{u} - d\bar{d}\right] \qquad \eta_s = s\bar{s}$$

The physical meson basis:

$$\eta, \qquad \eta', \qquad \pi^0$$

$$\begin{pmatrix} \pi^0 \\ \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} 1 & \beta + \psi cos\phi & -\psi sin\phi \\ -\psi - \beta cos\phi & cos\phi & -sin\phi \\ -\beta sin\phi & sin\phi & cos\phi \end{pmatrix} \begin{pmatrix} \eta_- \\ \eta_+ \\ \eta_s \end{pmatrix}$$

so

$$\eta_- = \pi^0 + (-\psi - \beta cos\phi)\eta + (-\beta sin\phi)\eta'$$

$$f_0(980) \to \pi^0(\eta_-, I = 1)\, \pi^0(\eta_-, I = 1) \to \pi^0\pi^0 + 2(-\psi - \beta cos\phi)\pi^0\eta + \cdots$$

$$sin\phi = \sqrt{\frac{\left(M_{\eta'}^2 - m_{ss}^2\right)\left(M_\eta^2 - M_{\pi^0}^2\right)}{\left(M_{\eta'}^2 - M_\eta^2\right)\left(m_{ss}^2 - M_{\pi^0}^2\right)}}$$

$$\beta = \frac{1}{2}\frac{m_{dd}^2 - m_{uu}^2}{M_{\eta'}^2 - M_{\pi^0}^2} + \frac{f_u - f_d}{f_u + f_d}$$

$$\psi = \frac{1}{2} cos\phi \frac{M_{\eta'}^2 - M_\eta^2}{M_{\eta'}^2 - M_{\pi^0}^2}\frac{m_{dd}^2 - m_{uu}^2}{M_\eta^2 - M_{\pi^0}^2}$$

$$\epsilon = \psi + \beta cos\phi$$

Here, ϵ is the $\pi^0 - \eta$ mixing angle and can be extracted from the decay processes includes $\pi^0 - \eta$ mixing :

$$\epsilon \sim 0.2$$

so

$$\frac{Br(f_0(980) \to \pi^0\eta)}{Br(f_0(980) \to \pi^0\pi^0)} = 4\epsilon^2 \frac{\lambda^{\frac{1}{2}}\left(m_{f_0(980)}^2, m_\eta^2, m_{\pi^0}^2\right)}{\lambda^{\frac{1}{2}}\left(m_{f_0(980)}^2, m_{\pi^0}^2, m_{\pi^0}^2\right)}$$

arXiv:0509031[hep-ph] P. Kroll

- $\theta = (36 \pm 2)^{\circ}$:

$$Br(D_s^+ \to f_0(980)e^+\nu_e \to \eta\pi e^+\nu_e) = \left(1.12^{+0.52}_{-0.47}\right) \times 10^{-6}$$

$$Br(D_s^+ \to a_0^0(980)e^+\nu_e \to \eta\pi e^+\nu_e) = \begin{cases} (2.04^{+1.18}_{-1.10}) \times 10^{-5}, & \xi_{fa} = (0.99 \pm 0.35) \times 10^{-2} \\ \left(8.43^{+6.45}_{-6.19}\right) \times 10^{-6}, & \xi_{fa} = (0.41 \pm 025) \times 10^{-2} \end{cases}$$

- $\theta = (148 \pm 6)^{\circ}$:

$$Br(D_s^+ \to f_0(980)e^+\nu_e \to \eta\pi e^+\nu_e) = \left(1.24^{+0.59}_{-0.54}\right) \times 10^{-6}$$

$$Br(D_s^+ \to a_0^0(980)e^+\nu_e \to \eta\pi e^+\nu_e) = \begin{cases} (2.24^{+1.32}_{-1.24}) \times 10^{-5}, & \xi_{fa} = (0.99 \pm 0.35) \times 10^{-2} \\ (9.27^{+7.16}_{-6.91}) \times 10^{-6}, & \xi_{fa} = (0.41 \pm 025) \times 10^{-2} \end{cases}$$

- The width of the $f_0(980)$ with $f_0(980) \to \pi^0\pi^0 \to \eta\pi^0$ is much larger than the $a_0^0(980)$ with $f_0(980) - a_0^0(980)$ mixing .

- The electromagnetic process $f_0(980) \to \gamma^*\gamma^* \to \eta\pi^0$ may interfere the signal and has much larger width??????

- The electromagnetic process$D_s^+ \to s\bar{s}e^+\nu_e \to \gamma^*\gamma^*e^+\nu_e \to a_0(980)e^+\nu_e$may interfere the signal and has much larger width??????

8. conclusion

- The branching ratio of $Br(D_s^+ \to a_0^0(980)e^+\nu_e \to \eta\pi e^+\nu_e)$ can reach 10^{-5} order, which is hopeful observed in experiment.

- The branching ratio of $Br(D_s^+ \to a_0^0(980)e^+\nu_e \to \eta\pi e^+\nu_e)$have large uncertainties in theory, the uncertainties are partly caused by the decay constant and the decay width of $f_0(980)$ and the mixing intensity ξ_{fa}, so the other methods or decay channels to determine ξ_{fa} are needed.

- the decay process $D_s^+ \to f_0(980)e^+\nu_e \to \eta\pi^0 e^+\nu_e$ may contaminate the signal of $f_0(980) - a_0^0(980)$ mixing.

Thank you ！

- As for $J/\psi \to \phi\, a_0(980) \to \phi\eta\pi^0$, the electromagnetic process and the $f_0(980) - a_0^0(980)$ mixing can interfere with each other, the width of $a_0^0(980)$ in the electromagnetic process is much larger.

the signal: $J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\pi^0\eta$

EM process $\begin{cases} J/\psi \to \gamma^* \to \phi\, a_0(980) \\ J/\psi \to K^* K Loop \to \phi\, a_0(980) \end{cases}$

$J/\psi \to \phi f_0(980) \to \phi\pi^0\pi^0 \to \phi\pi^0\eta$