Amplitude analysis of $D \to K\pi\pi\pi$ and $D_S^+ \to \pi^+\pi^0\eta$ at BESIII

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Outline

- Introduction
  - Important variables
  - D^0, D^+, and D_s Dataset
  - DTag and Branching Fraction
- Amplitude analysis of $D \to K\pi\pi\pi$
  - $K^-\pi^+\pi^+\pi^-$, $K_S\pi^+\pi^+\pi^-$, $K^-\pi^+\pi^0\pi^0$
- Amplitude analysis of $D_S^+ \to \pi^+\pi^0\eta$
- Summary

Important Variables

- Beam-Constrained Mass (M_{BC})

$$M_{\rm BC} = \sqrt{E_{\rm beam}^2 - |\vec{p}_D|^2}$$

M_{BC} peaks at D mass:
momentum conservation

- Energy Difference (ΔE)

$$\Delta E = E_D - E_{\rm beam}$$

ΔE peaks at zero:
energy conservation

Typically cut on ΔE, then fit to M_{BC} obtain yield

BESIII Data Taken near $D\bar{D}$ Threshold

- BEPCII collider: $e^+e^- \to \psi(3770) \to D\bar{D}$
- 2.9 fb^{-1} dataset at $\psi(3770)$ resonance

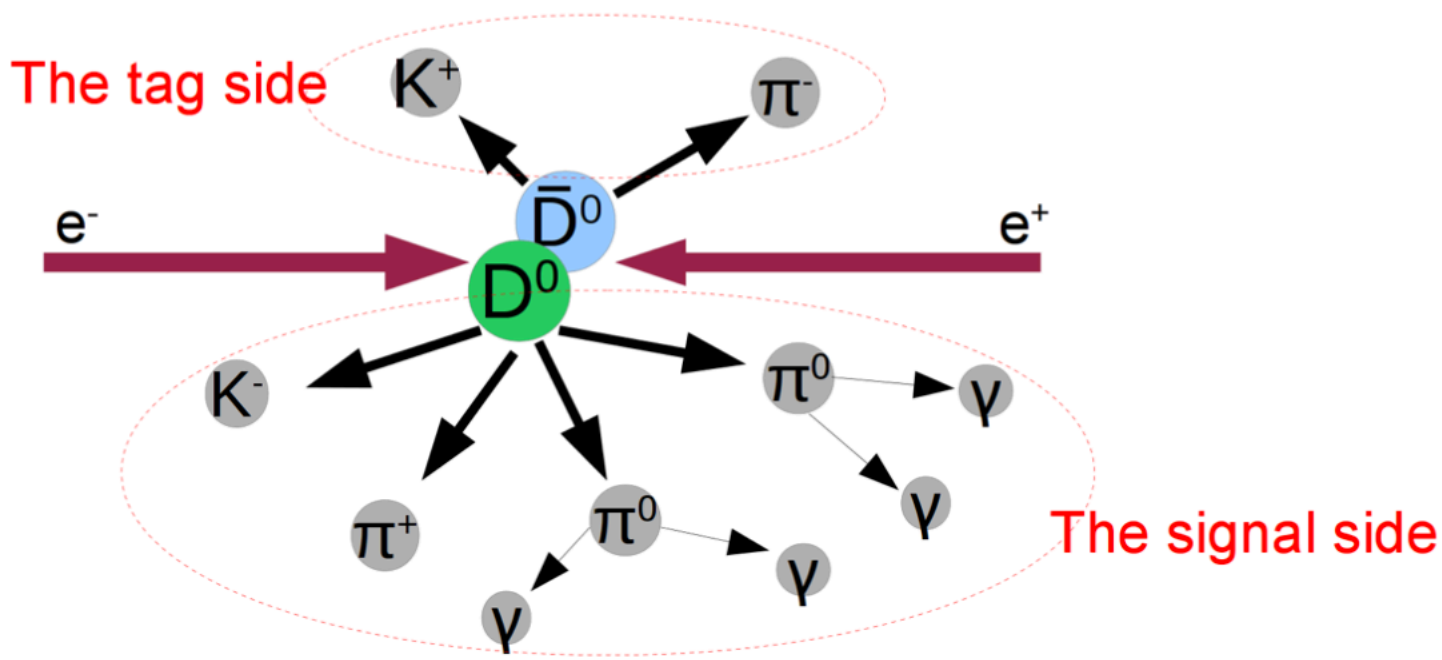
  M_{D0}= 1864.84 MeV M_{D+}= 1869.62 MeV

  $2M_{D0}$= 3729.68 MeV $2M_{D+}$= 3739.24 MeV

- New 3.19 fb^{-1} dataset at E_{cm} 4.178GeV
  - D_s are produced mostly via $e^+e^- \to D_sD_s^*$
- Advantages of $D\bar{D}$ pair production near threshold
  - The $D\bar{D}$ events are clean; not enough energy for even one additional pion
  - Tagging reduces background from light-quark "continuum" and other charm final states
  - Double tag technique can provide access to absolute BFs
  - Many systematic uncertainties cancel with tagging technique

DTag Technique

- There are two types of samples used in the Dtag technique: single tag (ST) and double tag (DT).
- Single tag: only one D meson is reconstructed through a chosen hadronic decay.
- Double tag: both D and $\bar{D}$ are reconstructed,
- the D reconstructed through the studied hadronic decay is called “the signal side”.
- the $\bar{D}$ reconstructed through well-known and clean hadronic decay modes is called “the tag side”.
- (Charge-conjugate states are implied throughout this talk.)



Branching Fraction and Tagging

- Single tag (ST)

$$N^{\mathrm{ST}}_{\mathrm{tag}} = 2N_{D^0\bar{D}^0}\mathcal{B}_{\mathrm{tag}}\varepsilon_{\mathrm{tag}}$$

- Double tag (DT)

$$N^{\mathrm{DT}}_{\mathrm{tag,sig}} = 2N_{D^0\bar{D}^0}\mathcal{B}_{\mathrm{tag}}\mathcal{B}_{\mathrm{sig}}\varepsilon_{\mathrm{tag,sig}}$$

$\varepsilon_{\mathrm{tag,sig}} \approx \varepsilon_{\mathrm{tag}}\varepsilon_{\mathrm{sig}}$ (factorization)

where $N_{D^0\bar{D}^0}$ is the total number of produced $D^0\bar{D}^0$ pairs, $\mathcal{B}_{\mathrm{tag(sig)}}$ is the branching fraction of the tag (signal) side, and the ε are the corresponding efficiencies.

$$\boxed{\mathcal{B}_{\mathrm{sig}} = \frac{N^{\mathrm{DT}}_{\mathrm{tag,sig}}}{N^{\mathrm{ST}}_{\mathrm{tag}}}\frac{\varepsilon_{\mathrm{tag}}}{\varepsilon_{\mathrm{tag,sig}}}}$$

$N_{D^0\bar{D}^0}$, $\mathcal{B}_{\mathrm{tag}}$ are canceled.
$\varepsilon_{\mathrm{tag}}$ is approximately canceled due to factorization

This is the basic idea for branching fraction.
Equations used in analysis vary case by case.

Amplitude Analysis of $K\pi\pi\pi$

- There are seven $D \to K3\pi$ modes:

  $D^0 \to K^-\pi^+\pi^+\pi^-$ (published on PRD) PhysRevD.95.072010

  $D^0 \to K^-\pi^+\pi^0\pi^0$ (expected to publish on PRD soon)

  $D^0 \to K_S\pi^0\pi^0\pi^0$

  $D^0 \to K_S\pi^+\pi^-\pi^0$ (on-going)

  $D^+ \to K^-\pi^+\pi^+\pi^0$ (on-going)

  $D^+ \to K_S\pi^+\pi^0\pi^0$ (on-going)

  $D^+ \to K_S\pi^+\pi^+\pi^-$ (expected to publish on PRD soon)

- Four-body decays are in five-dimensions
- We have
  - Partial Wave Analysis Tools based on CPU and GPU kernel
  - Great Electro-Magnetic Calorimeter (EMC) with CsI
    $\to$ superior resolution and efficiency of π^0
  - Largest dataset at $\psi(3770)$ resonance
    $\to$ small statistical errors and clean background

Motivation

- The measurements of the sub-modes in $D \to K\pi\pi\pi$ provides a window to study the decays $D \to AP$ and $D \to VV$ (A=axial-vector, V=vector), both of them are important in learning the CPV in charm decays but less effective experimental measurements.
- The knowledge of intermediate process can be widely used in many measurements, such as:
  - Branching fraction measurement
  - Strong phase measurement
  - CKM unitary triangle measurement
- $D^0 \to K^-\pi^+\pi^+\pi^-$ and $D^0 \to K^-\pi^+\pi^0\pi^0$ are very useful high-statistics modes and usually used to reconstruct neutral D mesons as tags. Their detail structures and branching fractions are important for charm studies using the tag technique.

Partial Wave Analysis

The Signal PDF

I am going to fit

$$A(a_i, p_j) = \sum_i a_i A_i(p_j)$$

$$S(a_i, p_j) = \frac{\epsilon(p_j)|A(a_i, p_j)|^2 R_4(p_j)}{\int \epsilon(p_j)|A(a_i, p_j)|^2 R_4(p_j) dp_j}$$

where p_j is the daughter particles' four momenta and a_i is the complex coefficient for amplitude modes. $\epsilon(p_j)$ is the efficiency parameterized in terms of the daughter particles' four momenta. R_4 is the 4-body phase space

$$A_i(p_j) = P_i^1(p_j) P_i^2(p_j) S_i(p_j) F_i^1(p_j) F_i^2(p_j) F_i^D(p_j)$$

where $F_i^D(p_j)$ is the Blatt-Weisskopf Barrier factor for D meson. $P_i^{1,2}(p_j)$ and $F_i^{1,2}(p_j)$ is the propagator and the Blatt-Weisskopf Barrier factor, respectively, of the two resonance states for the quasi-two-body type or of the first and the second resonance states for the cascade type. $S_i(p_j)$ is the spin factor. Finally, the likelihood can be defined as

For n events $\prod_{j=1}^{n} S(a_i, p_j)$

Define the likelihood $L = \prod_{j=1}^{n} S(a_i, p_j)$

Partial Wave Analysis

Independent of a_i

$$\ln L = \sum_{j}^{N_{selected}} \ln\left(\frac{|A(a_i, p_j)|^2 R_4(p_j)}{\int \epsilon(p_j)|A(a_i, p_j)|^2 R_4(p_j) dp_j}\right) + \sum_{j}^{N_{selected}} \ln \cancel{\epsilon}(p_j)$$

$$\int \epsilon(p_j)|A(a_i, p_j)|^2 R_4(p_j) dp_j \approx \frac{1}{N_{generated}} \sum_{j}^{N_{selected}} |A(a_i, p_j)|^2$$

Phase space MC sample can be used to deal with the MC integration.
We replace phase space MC sample by signal MC sample
for better precision.

$$\int \epsilon(p_j)|A(a_i, p_j)|^2 R_4(p_j) dp_j \approx \frac{1}{N_{MC}} \sum_{j}^{N_{MC}} \frac{|A(a_i, p_j)|^2}{|A(a_i^{gen}, p_j)|^2}$$

We further consider the effects of detector efficiency difference between data and MC simulation for pi0 reconstruction, PID, and tracking

$$\int \epsilon(p_j)|A(a_i, p_j)|^2 R_4(p_j) dp_j \approx \frac{1}{N_{MC}} \sum_{j}^{N_{MC}} \frac{|A(a_i, p_j)|^2 \gamma_\epsilon(p_j)}{|A(a_i^{gen}, p_j)|^2}$$

where $\gamma_\epsilon(p_j) = \prod_j \frac{\epsilon_{j,\mathrm{data}}(p_j)}{\epsilon_{j,\mathrm{MC}}(p_j)}$

Amplitude Analysis of $D^0 \to K^-\pi^+\pi^0\pi^0$

Event selection:

Reconstruct both of $D\bar{D}$ (Double tag) through

$D^0 \to K^-\pi^+\pi^0\pi^0$ (signal) vs. $\bar{D}^0 \to K^+\pi^-$ (tag)

$$M_{\rm BC} = \sqrt{E^2_{\rm beam} - |\vec{p}_D|^2}$$

$$\Delta E = E_D - E_{\rm beam}$$

Cuts on Tag: $1.8575 < M_{BC} < 1.8775$ GeV/c^2 $-0.03 < \Delta E < 0.02$ GeV

Cuts on signal: $1.8600 < M_{BC} < 1.8730$ GeV/c^2 $-0.04 < \Delta E < 0.02$ GeV

Peaking background: $K^-K_S\pi^+$ with $K_S \to \pi^0\pi^0$

A K_S mass veto is applied to reduce this peaking background from ~2% to 0.07%

Total event: 5950

Total background: ~ 1%

Fit: Unbinned likelihood fit with coherent summation of 26 amplitude modes, whose significances are > 4 σ

$$A_i(p_j) = P_i^1(p_j)P_i^2(p_j)S_i(p_j)F_i^1(p_j)F_i^2(p_j)F_i^D(p_j)$$

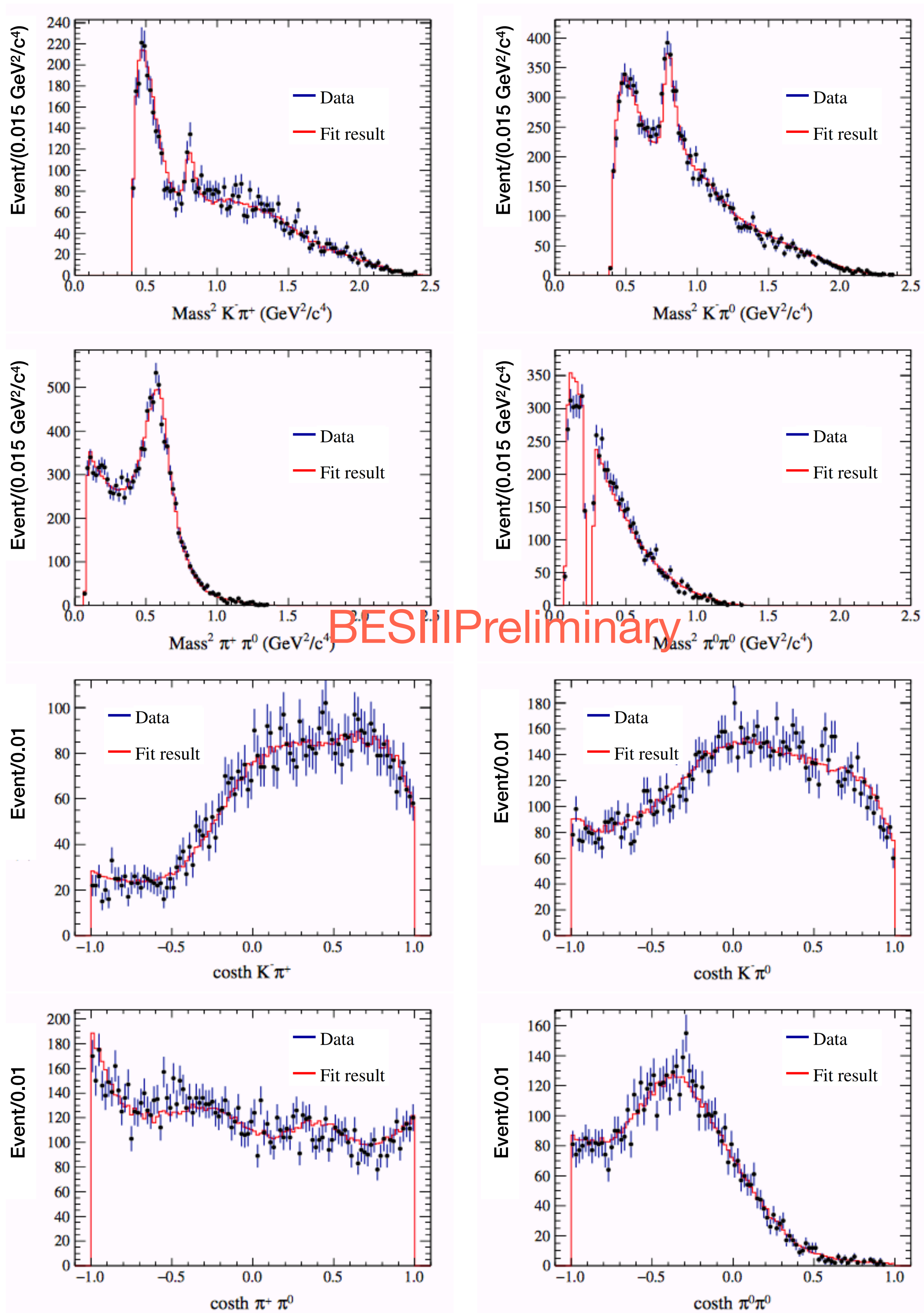
Amplitude **Propagator** **Spin factor** **Barrier factor**

Amplitude Analysis Results of $D^0 \to K^-\pi^+\pi^0\pi^0$

Amplitude mode	FF(%)	Phase (ϕ)
$D \to SS$		
$D \to (K^-\pi^+)_{S\text{-wave}}(\pi^0\pi^0)_S$	$6.92 \pm 1.44 \pm 2.86$	$-0.75 \pm 0.15 \pm 0.47$
$D \to (K^-\pi^0)_{S\text{-wave}}(\pi^+\pi^0)_S$	$4.18 \pm 1.02 \pm 1.77$	$-2.90 \pm 0.19 \pm 0.47$
$D \to AP, A \to VP$		
$D \to K^-a_1(1260)^+, \rho^+\pi^0[S]$	$28.36 \pm 2.50 \pm 3.53$	0 (fixed)
$D \to K^-a_1(1260)^+, \rho^+\pi^0[D]$	$0.68 \pm 0.29 \pm 0.30$	$-2.05 \pm 0.17 \pm 0.25$
$D \to K_1(1270)^-\pi^+, K^{*-}\pi^0[S]$	$0.15 \pm 0.09 \pm 0.18$	$1.84 \pm 0.34 \pm 0.43$
$D \to K_1(1270)^0\pi^0, K^{*0}\pi^0[S]$	$0.39 \pm 0.18 \pm 0.30$	$-1.55 \pm 0.20 \pm 0.26$
$D \to K_1(1270)^0\pi^0, K^{*0}\pi^0[D]$	$0.11 \pm 0.11 \pm 0.13$	$-1.35 \pm 0.43 \pm 0.48$
$D \to K_1(1270)^0\pi^0, K^-\rho^+[S]$	$2.71 \pm 0.38 \pm 0.29$	$-2.07 \pm 0.09 \pm 0.20$
$D \to (K^{*-}\pi^0)_A\pi^+, K^{*-}\pi^0[S]$	$1.85 \pm 0.62 \pm 1.11$	$1.93 \pm 0.10 \pm 0.15$
$D \to (K^{*0}\pi^0)_A\pi^0, K^{*0}\pi^0[S]$	$3.13 \pm 0.45 \pm 0.58$	$0.44 \pm 0.12 \pm 0.21$
$D \to (K^{*0}\pi^0)_A\pi^0, K^{*0}\pi^0[D]$	$0.46 \pm 0.17 \pm 0.29$	$-1.84 \pm 0.26 \pm 0.42$
$D \to (\rho^+K^-)_A\pi^0, K^-\rho^+[D]$	$0.75 \pm 0.40 \pm 0.60$	$0.64 \pm 0.36 \pm 0.53$
$D \to AP, A \to SP$		
$D \to ((K^-\pi^+)_{S\text{-wave}}\pi^0)_A\pi^0$	$1.99 \pm 1.08 \pm 1.55$	$-0.02 \pm 0.25 \pm 0.53$
$D \to VS$		
$D \to (K^-\pi^0)_{S\text{-wave}}\rho^+$	$14.63 \pm 1.70 \pm 2.41$	$-2.39 \pm 0.11 \pm 0.35$
$D \to K^{*-}(\pi^+\pi^0)_S$	$0.80 \pm 0.38 \pm 0.26$	$1.59 \pm 0.19 \pm 0.24$
$D \to K^{*0}(\pi^0\pi^0)_S$	$0.12 \pm 0.27 \pm 0.27$	$1.45 \pm 0.48 \pm 0.51$
$D \to VP, V \to VP$		
$D \to (K^{*-}\pi^+)_V\pi^0$	$2.25 \pm 0.43 \pm 0.45$	$0.52 \pm 0.12 \pm 0.17$
$D \to VV$		
$D[S] \to K^{*-}\rho^+$	$5.15 \pm 0.75 \pm 1.28$	$1.24 \pm 0.11 \pm 0.23$
$D[P] \to K^{*-}\rho^+$	$3.25 \pm 0.55 \pm 0.41$	$-2.89 \pm 0.10 \pm 0.18$
$D[D] \to K^{*-}\rho^+$	$10.90 \pm 1.53 \pm 2.36$	$2.41 \pm 0.08 \pm 0.16$
$D[P] \to (K^-\pi^0)_V\rho^+$	$0.36 \pm 0.19 \pm 0.27$	$-0.94 \pm 0.19 \pm 0.28$
$D[D] \to (K^-\pi^0)_V\rho^+$	$2.13 \pm 0.56 \pm 0.92$	$-1.93 \pm 0.22 \pm 0.25$
$D[D] \to K^{*-}(\pi^+\pi^0)_V$	$1.66 \pm 0.52 \pm 0.61$	$-1.17 \pm 0.20 \pm 0.39$
$D[S] \to (K^-\pi^0)_V(\pi^+\pi^0)_V$	$5.17 \pm 1.91 \pm 1.82$	$-1.74 \pm 0.20 \pm 0.31$
$D \to TS$		
$D \to (K^-\pi^+)_{S\text{-wave}}(\pi^0\pi^0)_T$	$0.30 \pm 0.21 \pm 0.32$	$-2.93 \pm 0.31 \pm 0.82$
$D \to (K^-\pi^0)_{S\text{-wave}}(\pi^+\pi^0)_T$	$0.14 \pm 0.12 \pm 0.10$	$2.23 \pm 0.38 \pm 0.65$

BESIIIPreliminary

Amplitude Analysis Results of $D^0 \rightarrow K^-\pi^+\pi^0\pi^0$



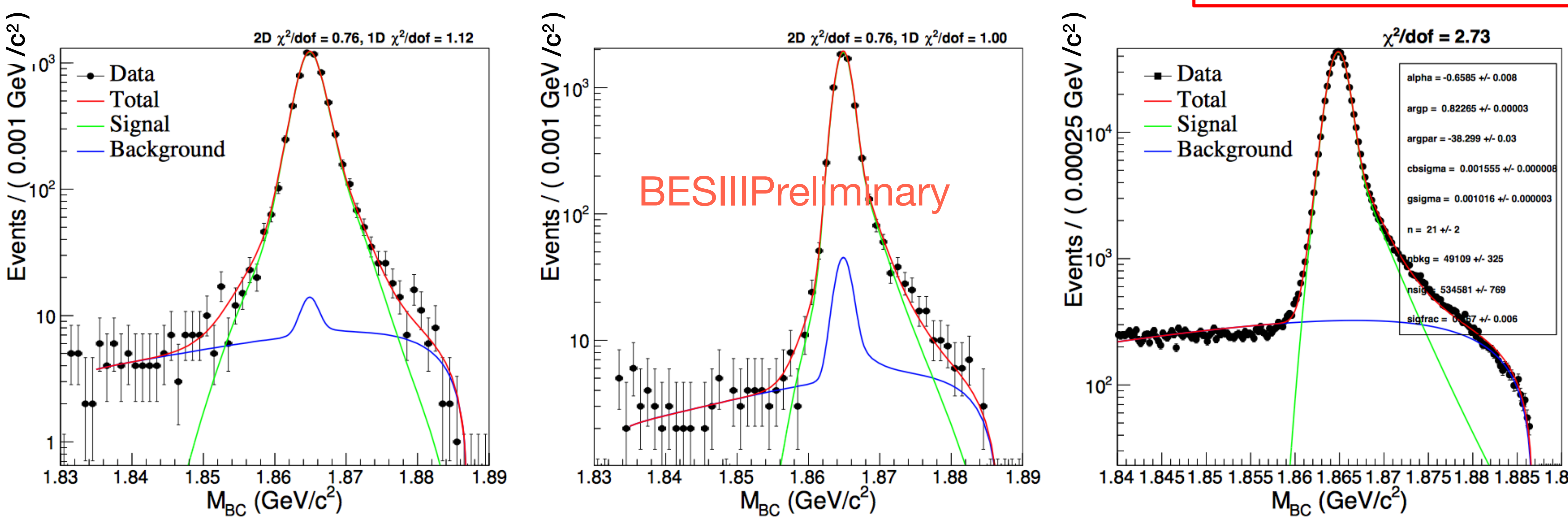
A mixed-sample method* is used to determine the goodness-of-fit of this five-dimensional unbinned likelihood fit.

***M. Williams, Journal of Instrumentation 5, P09004 (2010).**

Branching Fraction Results of $D^0 \to K^-\pi^+\pi^0\pi^0$

Double tag(DT) $D^0 \to K^-\pi^+\pi^0\pi^0$ vs. $\bar{D}^0 \to K^+\pi^-$
Single tag(ST) $\bar{D}^0 \to K^+\pi^-$

$$\boxed{\mathcal{B}_{\rm sig} = \frac{N^{\rm DT}_{\rm tag,sig}}{N^{\rm ST}_{\rm tag}} \frac{\varepsilon_{\rm tag}}{\varepsilon_{\rm tag,sig}}}$$



(a)DT $(K^-\pi^+\pi^0\pi^0)$ (b)DT $(K^+\pi^-)$ (c)ST

The amplitude analysis result is used to determine the detection efficiency, where the DT efficiency is 8.39%

The branching fraction is determined to be

$$\boxed{\mathcal{B}(D^0 \to K^-\pi^+\pi^0\pi^0) = (8.98 \pm 0.13({\rm stat}) \pm 0.40({\rm syst}))\%}$$

BESIIIPreliminary

Amplitude Analysis of $D^0 \to K^-\pi^+\pi^+\pi^-$

Event selection:

Double tag $D^0 \to K^-\pi^+\pi^+\pi^-$ vs. $\bar{D}^0 \to K^+\pi^-$

Peaking background:

$K^-K_S\pi^+$ with $Ks \to \pi^+\pi^-$ is the dominate background and peaks as the signal. Its number is estimated to be 96.8 +- 14.5 based on MC

Other background: < 10

The number of event selected is 15912 with a purity of 99.4%

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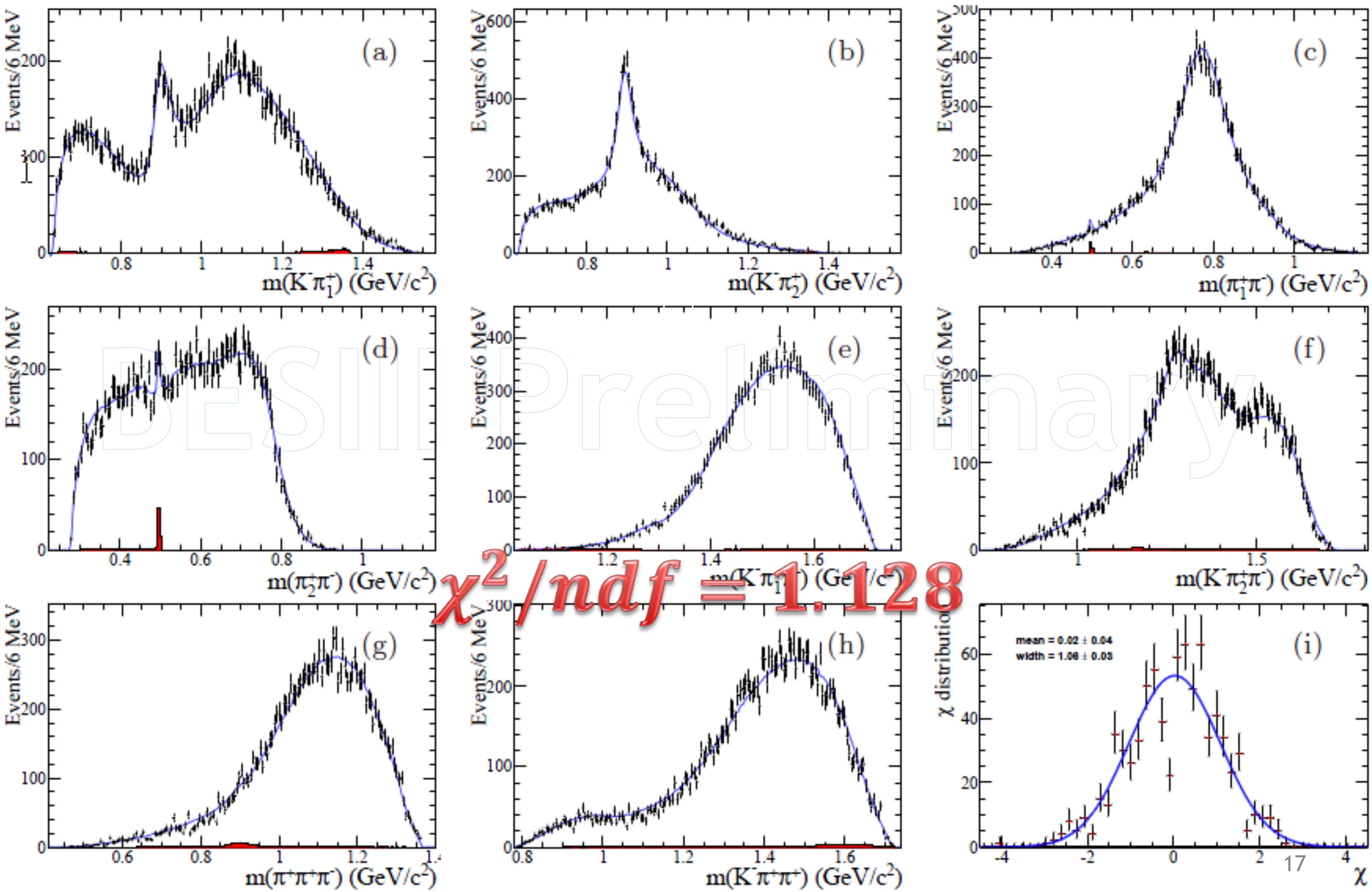
Amplitude Analysis Results of $D^0 \to K^-\pi^+\pi^+\pi^-$

Amplitude	ϕ_i	Fit fraction (%)
$D^0[S] \to \bar{K}^*\rho^0$	$2.35 \pm 0.06 \pm 0.18$	$6.5 \pm 0.5 \pm 0.8$
$D^0[P] \to \bar{K}^*\rho^0$	$-2.25 \pm 0.08 \pm 0.15$	$2.3 \pm 0.2 \pm 0.1$
$D^0[D] \to \bar{K}^*\rho^0$	$2.49 \pm 0.06 \pm 0.11$	$7.9 \pm 0.4 \pm 0.7$
$D^0 \to K^-a_1^+(1260)$, $a_1^+(1260)[S] \to \rho^0\pi^+$	0(fixed)	$53.2 \pm 2.8 \pm 4.0$
$D^0 \to K^-a_1^+(1260)$, $a_1^+(1260)[D] \to \rho^0\pi^+$	$-2.11 \pm 0.15 \pm 0.21$	$0.3 \pm 0.1 \pm 0.1$
$D^0 \to K_1^-(1270)\pi^+$, $K_1^-(1270)[S] \to \bar{K}^{*0}\pi^-$	$1.48 \pm 0.21 \pm 0.24$	$0.1 \pm 0.1 \pm 0.1$
$D^0 \to K_1^-(1270)\pi^+$, $K_1^-(1270)[D] \to \bar{K}^{*0}\pi^-$	$3.00 \pm 0.09 \pm 0.15$	$0.7 \pm 0.2 \pm 0.2$
$D^0 \to K_1^-(1270)\pi^+$, $K_1^-(1270) \to K^-\rho^0$	$-2.46 \pm 0.06 \pm 0.21$	$3.4 \pm 0.3 \pm 0.5$
$D^0 \to (\rho^0K^-)_{\mathrm{A}}\pi^+$, $(\rho^0K^-)_{\mathrm{A}}[D] \to K^-\rho^0$	$-0.43 \pm 0.09 \pm 0.12$	$1.1 \pm 0.2 \pm 0.3$
$D^0 \to (K^-\rho^0)_{\mathrm{P}}\pi^+$	$-0.14 \pm 0.11 \pm 0.10$	$7.4 \pm 1.6 \pm 5.7$
$D^0 \to (K^-\pi^+)_{\mathrm{S}}\rho^0$	$-2.45 \pm 0.19 \pm 0.47$	$2.0 \pm 0.7 \pm 1.9$
$D^0 \to (K^-\rho^0)_{\mathrm{V}}\pi^+$	$-1.34 \pm 0.12 \pm 0.09$	$0.4 \pm 0.1 \pm 0.1$
$D^0 \to (\bar{K}^{*0}\pi^-)_{\mathrm{P}}\pi^+$	$-2.09 \pm 0.12 \pm 0.22$	$2.4 \pm 0.5 \pm 0.5$
$D^0 \to \bar{K}^{*0}(\pi^+\pi^-)_{\mathrm{S}}$	$-0.17 \pm 0.11 \pm 0.12$	$2.6 \pm 0.6 \pm 0.6$
$D^0 \to (\bar{K}^{*0}\pi^-)_{\mathrm{V}}\pi^+$	$-2.13 \pm 0.10 \pm 0.11$	$0.8 \pm 0.1 \pm 0.1$
$D^0 \to ((K^-\pi^+)_{\mathrm{S}}\pi^-)_{\mathrm{A}}\pi^+$	$-1.36 \pm 0.08 \pm 0.37$	$5.6 \pm 0.9 \pm 2.7$
$D^0 \to K^-((\pi^+\pi^-)_{\mathrm{S}}\pi^+)_{\mathrm{A}}$	$-2.23 \pm 0.08 \pm 0.22$	$13.1 \pm 1.9 \pm 2.2$
$D^0 \to (K^-\pi^+)_{\mathrm{S}}(\pi^+\pi^-)_{\mathrm{S}}$	$-1.40 \pm 0.04 \pm 0.22$	$16.3 \pm 0.5 \pm 0.6$
$D^0[S] \to (K^-\pi^+)_{\mathrm{V}}(\pi^+\pi^-)_{\mathrm{V}}$	$1.59 \pm 0.13 \pm 0.41$	$5.4 \pm 1.2 \pm 1.9$
$D^0 \to (K^-\pi^+)_{\mathrm{S}}(\pi^+\pi^-)_{\mathrm{V}}$	$-0.16 \pm 0.17 \pm 0.43$	$1.9 \pm 0.6 \pm 1.2$
$D^0 \to (K^-\pi^+)_{\mathrm{V}}(\pi^+\pi^-)_{\mathrm{S}}$	$2.58 \pm 0.08 \pm 0.25$	$2.9 \pm 0.5 \pm 1.7$
$D^0 \to (K^-\pi^+)_{\mathrm{T}}(\pi^+\pi^-)_{\mathrm{S}}$	$-2.92 \pm 0.14 \pm 0.12$	$0.3 \pm 0.1 \pm 0.1$
$D^0 \to (K^-\pi^+)_{\mathrm{S}}(\pi^+\pi^-)_{\mathrm{T}}$	$2.45 \pm 0.12 \pm 0.37$	$0.5 \pm 0.1 \pm 0.1$

Amplitude Analysis Results of $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$

Projections of invariant mass (a-h) and χ distribution (i)



Amplitude Analysis Results of $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$

According to the intermediate resonances, we divides the 23 amplitudes into 7 subsets and call them components. Also, their fit fractions are calculated.

Fit fractions(FF) for different components

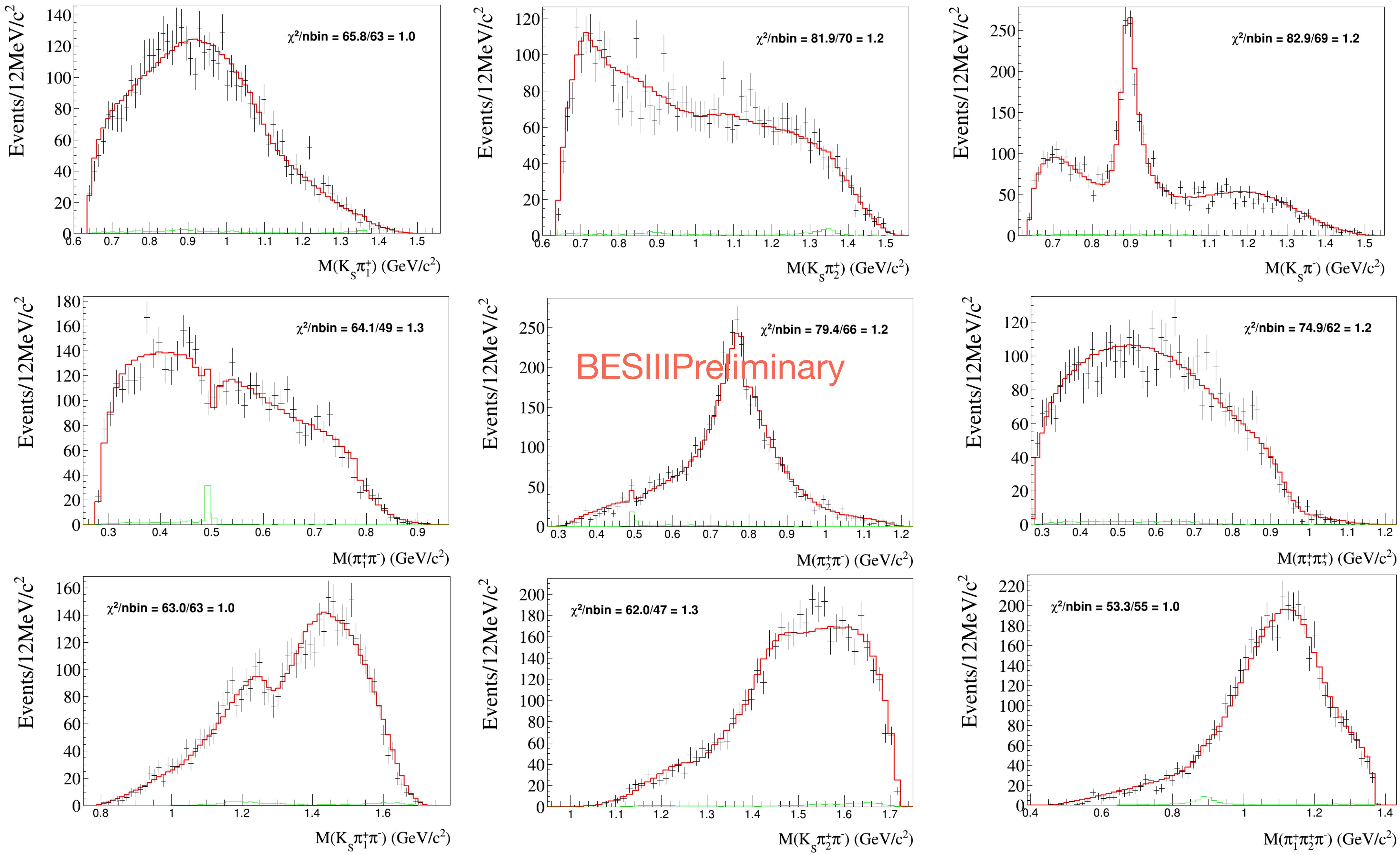
Component	Fit fraction (%)
$D^0 \to \bar{K}^{*0}\rho^0$	$12.3 \pm 0.4 \pm 0.5$
$D^0 \to K^- a_1^+(1260)(\rho^0\pi^+)$	$54.6 \pm 2.8 \pm 3.7$
$D^0 \to K_1^-(1270)(\bar{K}^{*0}\pi^-)\pi^+$	$0.8 \pm 0.2 \pm 0.2$
$D^0 \to K_1^-(1270)(K^-\rho^0)\pi^+$	$3.4 \pm 0.3 \pm 0.2$
$D^0 \to K^-\pi^+\rho^0$	$8.4 \pm 1.1 \pm 2.2$
$D^0 \to \bar{K}^{*0}\pi^+\pi^-$	$7.0 \pm 0.4 \pm 0.3$
$D^0 \to K^-\pi^+\pi^+\pi^-$	$21.9 \pm 0.6 \pm 0.6$

Amplitude Analysis of $D^+ \to K_S\pi^+\pi^+\pi^-$

The data can be described with 12 amplitudes with corresponding phases and fit fractions shown below:

Amplitude	ϕ	fit fraction
$D^+ \to K_S^0 a_1(1260)^+, a_1(1260)^+ \to \rho^0\pi^+[S]$	0.000(fixed)	$0.567 \pm 0.020 \pm 0.044$
$D^+ \to K_S^0 a_1(1260)^+, a_1(1260)^+ \to f_0(500)\pi^+$	$-2.023 \pm 0.068 \pm 0.113$	$0.050 \pm 0.006 \pm 0.007$
$D^+ \to \bar{K}_1(1400)^0\pi^+, \bar{K}_1(1400)^0 \to K^{*-}\pi^+[S]$	$-2.714 \pm 0.038 \pm 0.051$	$0.380 \pm 0.013 \pm 0.014$
$D^+ \to \bar{K}_1(1400)^0\pi^+, \bar{K}_1(1400)^0 \to K^{*-}\pi^+[D]$	$3.431 \pm 0.137 \pm 0.117$	$0.015 \pm 0.004 \pm 0.005$
$D^+ \to \bar{K}_1(1270)^0\pi^+, \bar{K}_1(1270)^0 \to K_S^0\rho^0[S]$	$-0.418 \pm 0.070 \pm 0.087$	$0.036 \pm 0.004 \pm 0.002$
$D^+ \to \bar{K}(1460)^0\pi^+, \bar{K}(1460)^0 \to K_S^0\rho^0$	$-1.850 \pm 0.120 \pm 0.223$	$0.014 \pm 0.004 \pm 0.003$
$D^+ \to (K_S^0\rho^0)_A[D]\pi^+$	$2.328 \pm 0.097 \pm 0.068$	$0.011 \pm 0.003 \pm 0.002$
$D^+ \to K_S^0(\rho^0\pi^+)_P$	$1.656 \pm 0.083 \pm 0.056$	$0.031 \pm 0.004 \pm 0.010$
$D^+ \to (K^{*-}\pi^+)_A[S]\pi^+$	$-4.321 \pm 0.047 \pm 0.073$	$0.132 \pm 0.011 \pm 0.011$
$D^+ \to (K^{*-}\pi^+)_A[D]\pi^+$	$0.989 \pm 0.158 \pm 0.229$	$0.013 \pm 0.004 \pm 0.004$
$D^+ \to (K_S^0(\pi^+\pi^-)_S)_A\pi^+$	$-2.935 \pm 0.060 \pm 0.125$	$0.051 \pm 0.004 \pm 0.003$
$D^+ \to ((K_S^0\pi^-)_S\pi^+)_P\pi^+$	$1.864 \pm 0.069 \pm 0.288$	$0.022 \pm 0.003 \pm 0.003$

Amplitude Analysis of $D^+ \to K_S\pi^+\pi^+\pi^-$



Amplitude Analysis of $D^+ \to K_S\pi^+\pi^+\pi^-$

With the fit fractions (FF) of every components and the branching fraction of $D^0 \to K^-\pi^+\pi^+\pi^-$, the branching fractions of the components is calculated with

$$B(Component) = FF(Componet)B(D^+ \to K_S^0\pi^+\pi^+\pi^-).$$

The results are listed in the table below:

Component	Branching fraction(%)
$D^+ \to K_S^0 a_1(1260)^+, a_1(1260)^+ \to \rho^0\pi^+$	$1.769 \pm 0.062 \pm 0.136 \pm 0.062$
$D^+ \to K_S^0 a_1(1260)^+, a_1(1260)^+ \to f_0(500)\pi^+$	$0.156 \pm 0.019 \pm 0.022 \pm 0.006$
$D^+ \to \bar{K}_1(1400)^0\pi^+, \bar{K}_1(1400)^0 \to K^{*-}\pi^+, K^{*-} \to K_S^0\pi^-$	$1.161 \pm 0.047 \pm 0.051 \pm 0.041$
$D^+ \to \bar{K}_1(1270)^0\pi^+, \bar{K}_1(1270)^0 \to K_S^0\rho^0$	$0.112 \pm 0.012 \pm 0.007 \pm 0.004$
$D^+ \to \bar{K}(1460)^0\pi^+, \bar{K}(1460)^0 \to K_S^0\rho^0$	$0.044 \pm 0.012 \pm 0.011 \pm 0.002$
$D^+ \to K_S^0\pi^+\rho^0$ three-body	$0.137 \pm 0.016 \pm 0.015 \pm 0.005$
$D^+ \to K^{*-}\pi^+\pi^+$ three-body, $K^{*-} \to K_S^0\pi^-$	$0.434 \pm 0.037 \pm 0.062 \pm 0.015$
$D^+ \to K_S^0\pi^+\pi^+\pi^-$ nonresonant	$0.231 \pm 0.016 \pm 0.024 \pm 0.008$

In the table, the first and second uncertainties of the branching fractions are statistical and systematic uncertainties from the fit fractions, respectively. The third errors are the uncertainties related to $B(D^+ \to K_S^0\pi^+\pi^+\pi^-)$ in PDG.

- We measure the sub-mode branching fractions in $D^+ \to K_S^0\pi^+\pi^+\pi^-$ decay, which will be helpful in understanding the $D \to AP$ decays.
- The measurements of the decays with $K_1(1270)$ and $K_1(1400)$ involved provide some experimental information in understanding the mixture of the two excited Kaons.

Amplitude Analysis of $D_S^+ \to \pi^+\pi^0\eta$

Event selected with double tag

Tag modes:

$$D_s^- \to K_S^0 K^-, D_s^- \to K^+K^-\pi^-, D_s^- \to K_S^0 K^-\pi^0, D_s^- \to K^+K^-\pi^-\pi^0,$$
$$D_s^- \to K_S^0 K^+\pi^-\pi^-, D_s^- \to \pi^-\eta_{\gamma\gamma}, D_s^- \to \pi^-\eta'_{\pi^+\pi^-\eta}$$

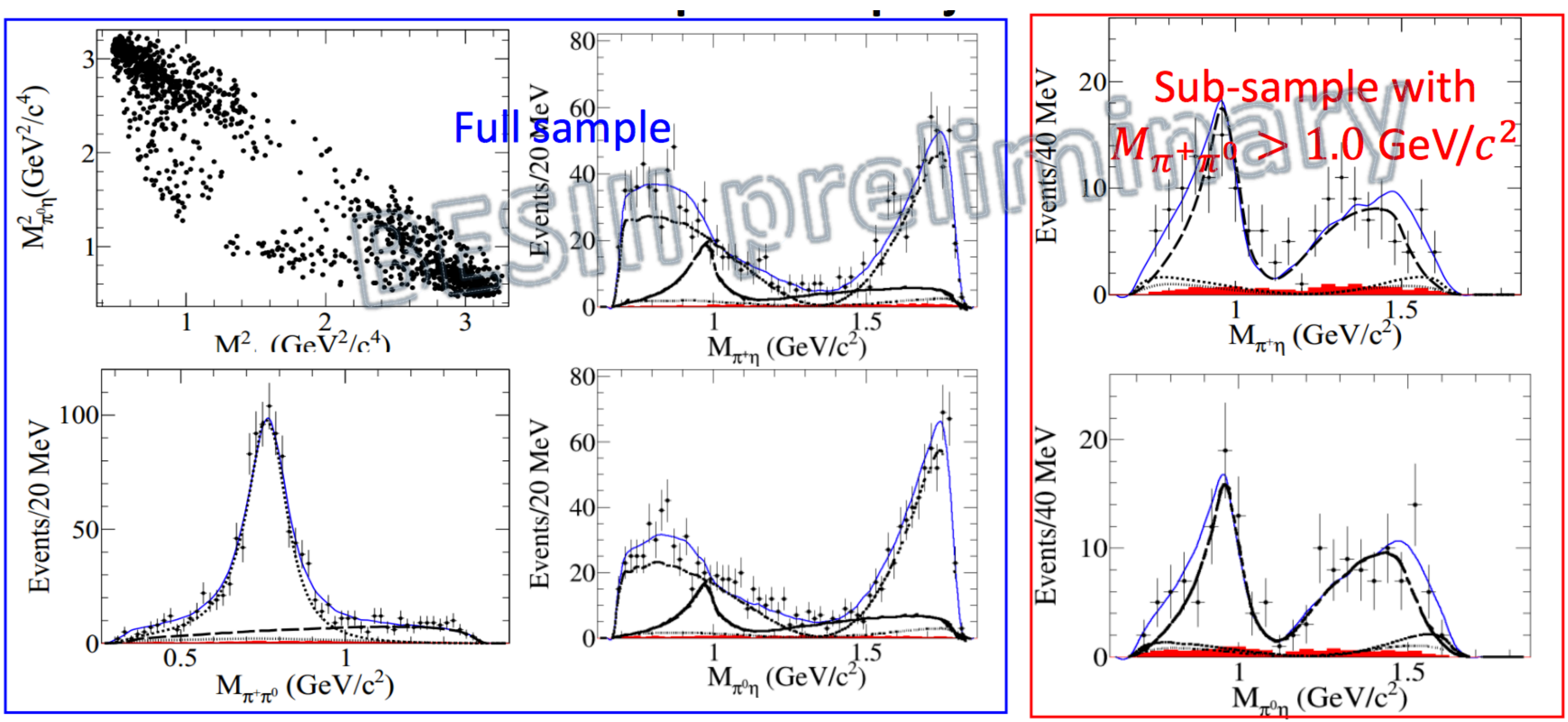
Data sample for amplitude analysis:

- A Multi-variate analysis is performed to suppress the background from fake η.
- The retained data sample has 1239 events with a purity of (97.7±0.5)%.

Amplitude Analysis of $D_s^+ \to \pi^+\pi^0\eta$

Observation of $D_s^+ \to a^0(980)^+\pi^0$

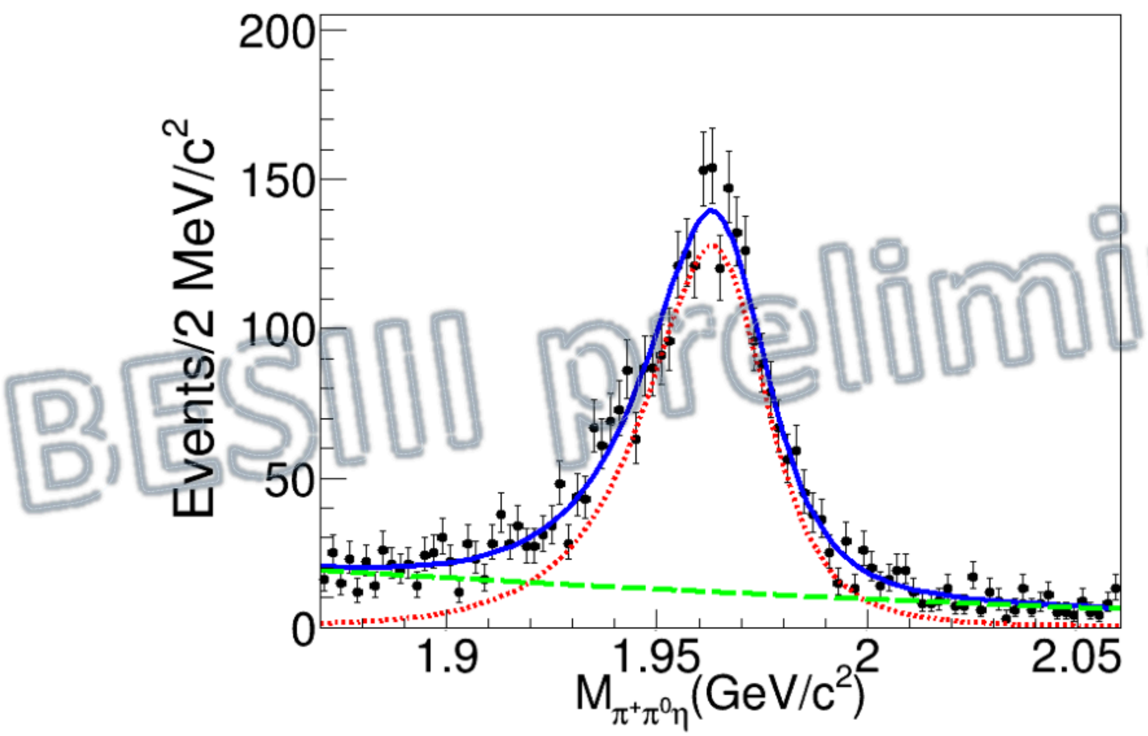
Amplitude	Significance (σ)	Phase	FF
$D_s^+ \to \rho^+\eta$	> 20	0.0 (fixed)	$0.783 \pm 0.050 \pm 0.021$
$D_s^+ \to (\pi^+\pi^0)_V\eta$	5.7	$0.612 \pm 0.172 \pm 0.342$	$0.054 \pm 0.021 \pm 0.026$
$D_s^+ \to a_0(980)\pi$	16.2	$2.794 \pm 0.087 \pm 0.041$	$0.232 \pm 0.023 \pm 0.034$



Dots with error bar: data; solid: total fit; dashed: $D_s^+ \to \rho^+\eta$; dotted: $D_s^+ \to (\pi^+\pi^0)_V\eta$; long dashed: $D_s^+ \to a_0(980)\pi$ (**with a Stat. significance of 16.2σ**).

Amplitude Analysis of $D_s^+ \to \pi^+\pi^0\eta$

Fit to signal mode



Total Tag yield: 255895 ± 1358.

DT yield: 2626 ± 77.

Efficiency is determined with the amplitude analysis result.

- Dots with error bars: data.
- Total fit.
- Signal: MC shape convoluted with a Gaussian.
- Background: second-order Chebychev.

$BF(D_s^+ \to \pi^+\pi^0\eta) = (9.50 \pm 0.28_{stat.} \pm 0.41_{sys.})\%$

Branching fraction (%) BESIII *Preliminary*
$\mathcal{B}(D_s^+ \to \rho^+\eta) = 7.44 \pm 0.48_{stat.} \pm 0.44_{sys.}$
$\mathcal{B}(D_s^+ \to a_0(980)\pi)^* = 2.20 \pm 0.22_{stat.} \pm 0.34_{sys.}$
$\mathcal{B}(D_s^+ \to a_0(980)^+\pi^0)^* = 1.46 \pm 0.15_{stat.} \pm 0.22_{sys.}$
$\mathcal{B}(D_s^+ \to a_0(980)^0\pi^+)^* = 1.46 \pm 0.15_{stat.} \pm 0.22_{sys.}$

BF(sub-mode n) = $\mathcal{B}(D_s^+ \to \pi^+\pi^0\eta)FF(n)$

First observation

- The measured $\mathcal{B}(D_s^+ \to a^0(980)^+\pi^0)$ is larger than other measured pure *W*-annihilation decays ($D_s^+ \to pn$, $D_s^+ \to w\pi^+$) by one order. This provides theoretical challenge in understanding such a large *W*-annihilation contribution in D→SP.

Summary

- DTag and DD^{bar} threshold data provides clean samples for amplitude analysis
- We have mature partial wave analysis tools based on CPU and GPU kernel
- Amplitude analysis of $K\pi\pi\pi$:
  - $D^0 \to K^-\pi^+\pi^+\pi^-$ is published in PRD95,072010
  - Preliminary results of $D^0 \to K^-\pi^+\pi^0\pi^0$ and $D^+ \to K_S\pi^+\pi^+\pi^-$ are obtained

    $\mathcal{B}(D^0 \to K^-\pi^+\pi^0\pi^0) = (8.98 \pm 0.13_{\mathrm{stat.}} \pm 0.40_{\mathrm{sys.}})\%$
  - Some $K\pi\pi\pi$ studies are on-going
- Preliminary results of $D_S^+ \to \pi^+\pi^0\eta$ are obtained
  - First observation of $D_S^+ \to a^0(980)^+\pi^0$
  - $\mathcal{B}(D_s^+ \to \pi^+\pi^0\eta) = (9.50 \pm 0.28_{stat.} \pm 0.41_{sys.})\%$