Improve measurement of decay parameters of Λ *in* $J/\psi \to \Lambda\bar{\Lambda}$

Jianyu Zhang, Haibo Li

Institute of High Energy Physics

@University Of Sicence And Technology Liaoning, Anshan
Aug 1$\sim$ 4, 2018

Outline

Motivation

A) The hyperon decay parameters, which characterize parity violation. However, in antihyperon nonleptonic decays, the uncertainties of the asymmetry parameters measured are still large.
$\alpha(\bar{\Lambda} \to \bar{p}\pi^{+}) = -0.71 \pm 0.08$ [?]

B) Searches for CP asymmetry in Λ nonleptonic decays have been previously performed, but the precision of the measurements are limited by statistics.

Derivation-Psionic form factors

The photon and the J/ψ are both vector particles, their corresponding annihilation processes will be similar. One can replace the form factors G_M and G_M in Fig(a) by the corresponding psionic form factors G_M^{ψ} and G_M^{ψ}.

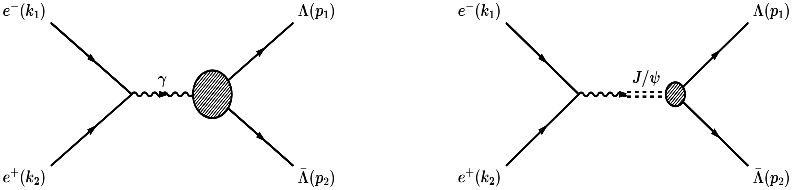

Figure: Graph describing the reaction $e^+e^- \to \bar{\Lambda}\Lambda$; a) genaral case, and b) mediated by the J/ψ resonance.

Derivation-Interaction vertex

We replace the electric charge e_{em} by a coupling strength e_ψ, which determined by the $J/\psi \to e^+e^-$ decay.

$$\Gamma^e_\mu(k_1, k_2) = -ie_\psi\gamma_\mu, \tag{1}$$

The J/ψ - hyperon vertex is:

$$\Gamma^\Lambda_\mu(p_1, p_2) = -ie_g\Big[G^\psi_M\gamma_\mu - \frac{2M}{Q^2}(G^\psi_M - G^\psi_E)Q_\mu\Big], \tag{2}$$

with $P = p_1 + p_2$, and $Q = p_1 - p_2$, and M the Lambda mass. The argument of the form factors equals $s = P^2$. The coupling strength e_g is determined by the hadronic-decay rate for $J/\psi \to \Lambda\bar{\Lambda}$.

Derivation-Construct parameters

We use combinations of form factors called D, α, and $\Delta\Phi$ to rewrite equation. The strength of form factors is measured by $D(s)$,

$$D(s) = s\left|G_M^{\psi}\right|^2 + 4M^2\left|G_E^{\psi}\right|^2. \tag{3}$$

a factor that multiplies all cross-section distributions. The ratio of form factors is measured by α,

$$\alpha = \frac{s\left|G_M^{\psi}\right|^2 - 4M^2\left|G_E^{\psi}\right|^2}{s\left|G_M^{\psi}\right|^2 + 4M^2\left|G_E^{\psi}\right|^2}, \tag{4}$$

with α satisfying $-1 \leq \alpha \leq 1$. The relative phase of form factors is measured by $\Delta\Phi$,

$$\frac{G_E^{\psi}}{G_M^{\psi}} = e^{i\Delta\Phi}\left|\frac{G_E^{\psi}}{G_M^{\psi}}\right|. \tag{5}$$

Derivation-J/ψ propagator

The propagator of J/ψ takes the form

$$\frac{g_{\mu\nu} - P_\mu P_\nu / m_\psi^2}{s - m_\psi^2 + i m_\psi \Gamma(\psi)}, \tag{6}$$

Since the J/ψ couples to conserved lepton and hyperon currents, the contribution from the $P_\mu P_\nu$ term vanishes. Thus we make the replacement

$$\frac{e_\psi e_g}{s - m_\psi^2 + i m_\psi \Gamma(\psi)} \to \frac{e_{em}^2}{s}, \tag{7}$$

where e_{em} is the electric charge.

Derivation-Cross section for $e^+e^- \to \Lambda(s_1)\bar{\Lambda}(s_2)$

The cross-section distribution for e^+e^- annihilation into polarized hyperons.

$$d\sigma = \frac{1}{2s}\,\mathcal{K}_\psi\,|\mathcal{M}_{red}|^2\ \mathrm{dLips}(k_1 + k_2; p_1, p_2), \tag{8}$$

with dLips the phase-space factor, with $s = P^2$, and with

$$\mathcal{K}_\psi = \frac{e_\psi^2 e_g^2}{(s - m_\psi^2)^2 + m_\psi^2\Gamma^2(m_\psi)}. \tag{9}$$

$$\left|\mathcal{M}_{red}(e^+e^- \to \Lambda(s_1)\overline{\Lambda}(s_2))\right|^2 = L_{\nu\mu}(k_1, k_2)\cdot K_{\nu\mu}(s_1, s_2) \tag{10}$$

with $L(k_1, k_2)$ and $K(p_1, p_2; s_1, s_2)$ lepton and hadron tensors, and s_1 and s_2 hyperon spin four-vectors.

Derivation-Lepton and hadron tensor

Lepton tensor including averages over lepton spins;

$$L_{\nu\mu}(k_1,k_2) = \frac{1}{4}\mathrm{Tr}\left[\gamma_\nu \not{k}_1 \gamma_\mu \not{k}_2\right] = k_{1\nu}k_{2\mu} + k_{2\nu}k_{1\mu} - \frac{1}{2}sg_{\nu\mu}. \quad (11)$$

Hadron tensor for polarized hyperons;

$$K_{\nu\mu}(s_1,s_2) = \mathrm{Tr}\Big[\overline{\Gamma}^{\Lambda}_{\nu}(\not{p}_1 + M)\frac{1}{2}(1+\gamma_5\not{s}_1) \times \Gamma^{\Lambda}_{\mu}(\not{p}_2 - M)\frac{1}{2}(1+\gamma_5\not{s}_2)\Big]/e_g^2, \quad (12)$$

Derivation-Spin four-vector

The spin four-vector $s(\mathbf{p},\mathbf{n})$ of a hyperon of mass M, three-momentum $\mathbf{p}$, and spin direction $\mathbf{n}$ in its rest system, is

$$s(\mathbf{p},\mathbf{n}) = \frac{n_{\parallel}}{M}(|\mathbf{p}|; E\hat{\mathbf{p}}) + (0; \mathbf{n}_{\perp}). \tag{13}$$

$n_{\parallel} = \mathbf{n}\cdot\hat{\mathbf{p}}$ and $\mathbf{n}_{\perp} = \mathbf{n} - \hat{\mathbf{p}}(\mathbf{n}\cdot\hat{\mathbf{p}})$. The four-vectors p and s are orthogonal. In the global c.m. system, three-momenta $\mathbf{p}$ and $\mathbf{k}$ are defined such that

$$\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}, \tag{14}$$

$$\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}, \tag{15}$$

and scattering angle by,

$$\cos\theta = \hat{\mathbf{p}}\cdot\hat{\mathbf{k}}. \tag{16}$$

Derivation-Cross section for $e^+e^- \to \Lambda(s_1)\bar{\Lambda}(s_2)$

The matrix element in Eq.(11) can be written as a sum of four terms

$$\left|\mathcal{M}_{red}(e^+e^- \to \Lambda(s_1)\overline{\Lambda}(s_2))\right|^2 = sD(s)\Big[H^{00}(0,0) + H^{05}(\mathbf{n}_1,0) + H^{50}(0,\mathbf{n}_2) + H^{55}(\mathbf{n}_1,\mathbf{n}_2)\Big]. \quad (17)$$

$$H^{00} = \mathcal{R} \quad (18)$$

$$H^{05} = \mathcal{S}\left[\frac{1}{\sin\theta}(\hat{\mathbf{p}}\times\hat{\mathbf{k}})\cdot\mathbf{n}_1\right] \quad (19)$$

$$H^{50} = \mathcal{S}\left[\frac{1}{\sin\theta}(\hat{\mathbf{p}}\times\hat{\mathbf{k}})\cdot\mathbf{n}_2\right] \quad (20)$$

$$H^{55} = \Big\{\mathcal{T}_1\mathbf{n}_1\cdot\hat{\mathbf{p}}\mathbf{n}_2\cdot\hat{\mathbf{p}} + \mathcal{T}_2\mathbf{n}_{1\perp}\cdot\mathbf{n}_{2\perp} + \mathcal{T}_3\mathbf{n}_{1\perp}\cdot\hat{\mathbf{k}}\mathbf{n}_{2\perp}\cdot\hat{\mathbf{k}} + \mathcal{T}_4\left(\mathbf{n}_1\cdot\hat{\mathbf{p}}\mathbf{n}_{2\perp}\cdot\hat{\mathbf{k}} + \mathbf{n}_2\cdot\hat{\mathbf{p}}\mathbf{n}_{1\perp}\cdot\hat{\mathbf{k}}\right)\Big\} \quad (21)$$

Derivation-Cross section for $e^+e^- \to \Lambda(s_1)\bar{\Lambda}(s_2)$

$$
\begin{aligned}
\mathcal{R} &= 1+\alpha\cos^2\theta, && (22)\\
\mathcal{S} &= \sqrt{1-\alpha^2}\sin\theta\cos\theta\sin(\Delta\Phi), && (23)\\
\mathcal{T}_1 &= \alpha+\cos^2\theta, && (24)\\
\mathcal{T}_2 &= -\alpha\sin^2\theta, && (25)\\
\mathcal{T}_3 &= 1+\alpha, && (26)\\
\mathcal{T}_4 &= \sqrt{1-\alpha^2}\cos\theta\cos(\Delta\Phi). && (27)
\end{aligned}
$$

Transverse components $\mathbf{n}_{1\perp}$ and $\mathbf{n}_{2\perp}$ are orthogonal to the Lambda hyperon momentum $\mathbf{p}$ in the global c.m. system.

Derivation-Polarization

The Lambda-hyperon polarization is obtained from Eq.(27) which shows that the polarization is directed along the normal to the scattering plane, $\hat{\mathbf{p}} \times \hat{\mathbf{k}}$, and that the value of the polarization is

$$P_\Lambda(\theta) = \frac{\mathcal{S}}{\mathcal{R}} = \frac{\sqrt{1-\alpha^2}\cos\theta\sin\theta}{1+\alpha\cos^2\theta}\sin(\Delta\Phi) \tag{28}$$

Derivation-Folding of distributions

Hyperon-decay distributions are obtained by a folding calculation,

$$|\mathcal{M}|^2 = \sum_{\pm s_1, \pm s_2} \Big\langle \big|\mathcal{M}(e^+e^- \to \Lambda(s_1)\bar{\Lambda}(s_2))\big|^2 \times \big|\mathcal{M}(\Lambda(s_1) \to p\pi^-)\big|^2 \big|\mathcal{M}(\bar{\Lambda}(s_2) \to \bar{p}\pi^+)\big|^2 \Big\rangle_{\mathbf{n}_1\mathbf{n}_2} . \quad (29)$$

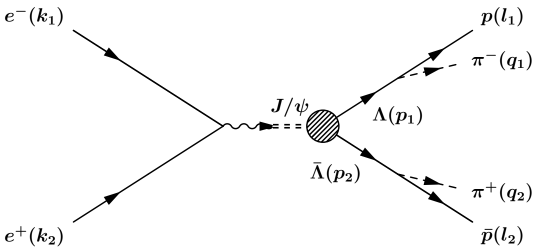

Figure: Graph describing the reaction $e^+e^- \to \Lambda(\to p\pi^-)\bar{\Lambda}(\to \bar{p}\pi^+)$.

Derivation-Folding of distributions

Production and decay distributions are,

$$\left|\mathcal{M}(e^+e^- \to \Lambda(s_1)\bar{\Lambda}(s_2))\right|^2 = L \cdot K(s_1, s_2), \tag{30}$$

$$\left|\mathcal{M}(\Lambda(s_1) \to p\pi^-)\right|^2 = R_\Lambda \left[1 - \alpha_1 l_1 \cdot s_1 / l_\Lambda\right], \tag{31}$$

$$\left|\mathcal{M}(\bar{\Lambda}(s_2) \to \bar{p}\pi^+)\right|^2 = R_\Lambda \left[1 - \alpha_2 l_2 \cdot s_2 / l_\Lambda\right], \tag{32}$$

with l_Λ the decay momentum in the Lambda rest system. R_Λ is determined by the Lambda decay rate.

Derivation-Folding of distributions

We evaluate the hyperon-decay distributions in the hyperon-rest systems, where

$$\left|\mathcal{M}(\Lambda(s_1) \to p\pi^-)\right|^2 = R_\Lambda \left[1 + \alpha_1 \hat{\mathbf{l}}_1 \cdot \mathbf{n}_1\right], \tag{33}$$

$$\left|\mathcal{M}(\bar{\Lambda}(s_2) \to \bar{p}\pi^+)\right|^2 = R_\Lambda \left[1 + \alpha_2 \hat{\mathbf{l}}_2 \cdot \mathbf{n}_2\right], \tag{34}$$

$\hat{\mathbf{l}}_1 = \mathbf{l}_1 / l_\Lambda$ is the unit vector in the direction of the proton momentum in the Lambda-rest system. Angular averages in Eq.(29) are calculated according to the prescription.

$$\left\langle (\mathbf{n} \cdot \mathbf{l})\mathbf{n} \right\rangle_{\mathbf{n}} = \mathbf{l}. \tag{35}$$

Derivation-Folding of distributions

The folding of the production distributions, Eqs.(18-21), with the decay distributions, Eqs.(33-34), yields

$$|\mathcal{M}_{red}|^2 = sD(s)R_\Lambda^2\Big[G^{00} + G^{05} + G^{50} + G^{55}\Big], \tag{36}$$

with the G^{ab} functions defined as

$$\begin{aligned}
G^{00} &= \mathcal{R}, && (37)\\
G^{05} &= \alpha_1\mathcal{S}\left[\frac{1}{\sin\theta}(\hat{\mathbf{p}}\times\hat{\mathbf{k}})\cdot\hat{\mathbf{l}}_1\right], && (38)\\
G^{50} &= \alpha_2\mathcal{S}\left[\frac{1}{\sin\theta}(\hat{\mathbf{p}}\times\hat{\mathbf{k}})\cdot\hat{\mathbf{l}}_2\right], && (39)\\
G^{55} &= \alpha_1\alpha_2\Big\{\mathcal{T}_1\hat{\mathbf{l}}_1\cdot\hat{\mathbf{p}}\hat{\mathbf{l}}_2\cdot\hat{\mathbf{p}} + \mathcal{T}_2\hat{\mathbf{l}}_{1\perp}\cdot\hat{\mathbf{l}}_{2\perp} + \mathcal{T}_3\hat{\mathbf{l}}_{1\perp}\cdot\hat{\mathbf{k}}\hat{\mathbf{l}}_{2\perp}\cdot\hat{\mathbf{k}}\\
&\qquad + \mathcal{T}_4\Big(\hat{\mathbf{l}}_1\cdot\hat{\mathbf{p}}\hat{\mathbf{l}}_{2\perp}\cdot\hat{\mathbf{k}} + \hat{\mathbf{l}}_2\cdot\hat{\mathbf{p}}\hat{\mathbf{l}}_{1\perp}\cdot\hat{\mathbf{k}}\Big)\Big\}. && (40)
\end{aligned}$$

Derivation-Cross section for $e^+e^- \to \Lambda(\to p\pi^-)\bar{\Lambda}(\to \bar{p}\pi^+)$

From the general expression, we get

$$d\sigma = \frac{1}{64\pi^2}\frac{p}{k}\frac{\alpha_g\alpha_\psi}{(s-m_\psi^2)^2+m_\psi^2\Gamma^2(\psi)}\frac{\Gamma_\Lambda\Gamma_{\bar{\Lambda}}}{\Gamma^2(M)} \cdot \cdot \left(D(s)\sum_{a,b}G^{ab}\right)\Omega_\Lambda\Omega_1\Omega_2, \tag{41}$$

with k and p the initial- and final-state momenta; Ω_Λ the hyperon scattering angle in the global c.m. system; Ω_1 and Ω_2 decay angles measured in the rest systems of Λ and $\bar{\Lambda}$; Γ_Λ and $\Gamma_{\bar{\Lambda}}$ channel widths; and $\Gamma(M)$ and $\Gamma(\psi)$ total widths.

Derivation-Differential distributions

We choose a right-handed coordinate system with basis vectors

$$\mathbf{e}_z = \hat{\mathbf{p}}, \tag{42}$$
$$\mathbf{e}_y = \frac{1}{\sin\theta}(\hat{\mathbf{p}} \times \hat{\mathbf{k}}), \tag{43}$$
$$\mathbf{e}_x = \frac{1}{\sin\theta}(\hat{\mathbf{p}} \times \hat{\mathbf{k}}) \times \hat{\mathbf{p}}. \tag{44}$$

Expressed in terms of them the initial-state momentum

$$\hat{\mathbf{k}} = \sin\theta\, \mathbf{e}_x + \cos\theta\, \mathbf{e}_z. \tag{45}$$

The components of the unit vector in direction of the decay-proton momentum (in the Lambda rest system) are

$$\hat{\mathbf{l}}_1 = (\cos\phi_1 \sin\theta_1, \sin\phi_1 \sin\theta_1, \cos\theta_1), \tag{46}$$

$$\hat{\mathbf{l}}_{1\perp} = (\cos\phi_1 \sin\theta_1, \sin\phi_1 \sin\theta_1, 0). \tag{47}$$

Derivation-Differential distributions

The differential-cross-section distribution

$$d\sigma \propto \mathcal{W}(\boldsymbol{\xi})\, d\cos\theta \; d\Omega_1 d\Omega_2.$$

The differential-distribution function $\mathcal{W}(\boldsymbol{\xi})$ can be expressed as,

$$\begin{aligned} \mathcal{W}(\boldsymbol{\xi}) =& \mathcal{F}_0(\boldsymbol{\xi}) + \alpha\mathcal{F}_5(\boldsymbol{\xi}) \\ &+ \alpha_1\alpha_2 \left(\mathcal{F}_1(\boldsymbol{\xi}) + \sqrt{1-\alpha^2}\cos(\Delta\Phi)\mathcal{F}_2(\boldsymbol{\xi}) + \alpha\mathcal{F}_6(\boldsymbol{\xi})\right) \\ &+ \sqrt{1-\alpha^2}\sin(\Delta\Phi)\left(\alpha_1\mathcal{F}_3(\boldsymbol{\xi}) + \alpha_2\mathcal{F}_4(\boldsymbol{\xi})\right), \end{aligned} \tag{48}$$

Derivation-Differential distributions

Using a set of seven angular functions $\mathcal{F}_k(\boldsymbol{\xi})$ defined as:

$$\begin{aligned}
\mathcal{F}_0(\boldsymbol{\xi}) =& 1\\
\mathcal{F}_1(\boldsymbol{\xi}) =& \sin^2\theta \sin\theta_1 \sin\theta_2 \cos\phi_1 \cos\phi_2 + \cos^2\theta \cos\theta_1 \cos\theta_2\\
\mathcal{F}_2(\boldsymbol{\xi}) =& \sin\theta \cos\theta \left(\sin\theta_1 \cos\theta_2 \cos\phi_1 + \cos\theta_1 \sin\theta_2 \cos\phi_2\right)\\
\mathcal{F}_3(\boldsymbol{\xi}) =& \sin\theta \cos\theta \sin\theta_1 \sin\phi_1\\
\mathcal{F}_4(\boldsymbol{\xi}) =& \sin\theta \cos\theta \sin\theta_2 \sin\phi_2\\
\mathcal{F}_5(\boldsymbol{\xi}) =& \cos^2\theta\\
\mathcal{F}_6(\boldsymbol{\xi}) =& \cos\theta_1 \cos\theta_2 - \sin^2\theta \sin\theta_1 \sin\theta_2 \sin\phi_1 \sin\phi_2.
\end{aligned} \tag{49}$$

Reference

[1] G. Fldt and A. Kupsc, Phys. Lett. B **772**, 16 (2017) doi:10.1016/j.physletb.2017.06.011 [arXiv:1702.07288 [hep-ph]].

[2] G. Fäldt, Eur. Phys. J. A **52**, 141 (2016).

[3] G. Fäldt, Eur. Phys. J. A **51**, 74 (2015).