DECAY CONSTANTS AND DISTRIBUTION AMPLITUDES OF B MESON IN THE RELATIVISTIC POTENTIAL MODEL

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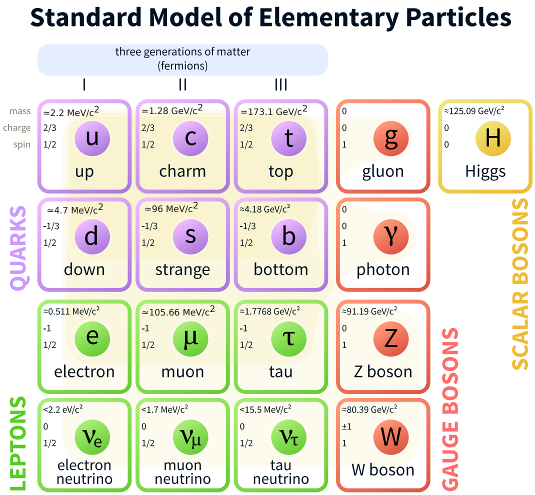
Outline

1. Introduction
2. Relativistic Potential Model and B Meson Wave Function
3. Decay Constants of B Mesons
4. Distribution Amplitudes of B Mesons
5. Application to Purely Leptonic B Decay
6. Summary and Outlook

Outline

The Standard Model

- **SM and NP**
  - **ATLAS, $t\bar{t}H$ production**
  - ν_l, R_K, R_{K^*}, **etc**
- **EW and QCD**
  - **QED,** $\alpha_e \simeq \frac{1}{137.036}$
  - **Weak interaction,** $G_F \simeq 1.166 \times 10^{-5}\mathbf{GeV}^{-2}$
  - **QCD coupling 'constant',** $\alpha_s(m_Z) \simeq 0.1181$



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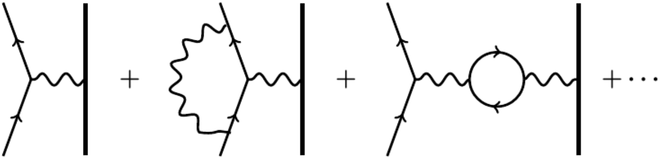
Perturbation Theory

- **In a perturbatively calculable theory, like QED, many processes can be predicted theoretically order by order to achieve very high accuracy.**



- **However, due to the running coupling constant $\alpha_s(\mu)$ and quark confinement, analysis of QCD-related processes must take non-perturbative contributions into consideration.**

Beauty Quark Modes

- The production, decay channels, and exotic states relevant with beauty quark are extensively and accurately measured by high energy experiments, providing us plenty of data used as criteria for the validation of theoretical research.
  - D0, BaBar, Belle, LHCb, etc
  - Belle II
- The wave functions describing internal interactions and structures of bound states, the decay constants and form factors presenting key information of hadronization and decay processes require our study into the contributions from both perturbative and non-perturbative regions.

Approches

Mass Spectra $\longleftrightarrow$ Wave Functions $\longrightarrow$ Decay Constants ...

- QCD Sum Rules
- Bethe-Salpeter equation
- Field Correlator Method
- Soft-wall Holographic
- Potential models
- Lattice QCD
- ...

Relativistic Potential Model

Why we choose relativistic potential model for heavy-light mesons:

- **concise forms**
- **relatively simple calculations**
- **no extra resources**
- **clear and intuitive physics picture**

The wave function from solving the Schrödinger wave equation with relativistic dynamics

$$(H_0 + H')\Psi(\vec{r}) = E\Psi(\vec{r})$$

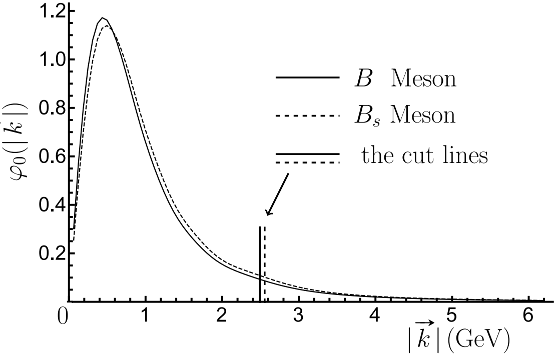
$$H_0 = \sqrt{\vec{k}_1^2 + m_1^2} + \sqrt{\vec{k}_2^2 + m_2^2} + V(r), \quad V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} + br + c$$

Wave Functions

With conventions for normalization factors, we obtain wave functions for *B* mesons in momentum space:

$$\int \mathrm{d}^3k \left|\Psi_0(\vec{k})\right|^2 = 1$$

$$\Psi_0(\vec{k}) = \frac{\varphi_0(\vec{k})}{|\vec{k}|} Y_{00}(\theta, \phi)$$

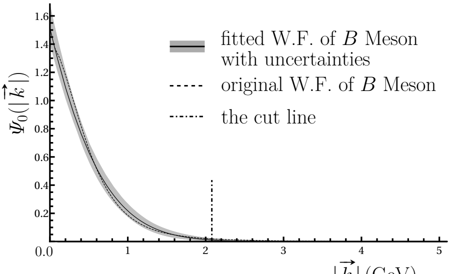


Applying them in meson-mass-spectra calculation, we get results consistent with Expt.

Fitted Forms

In practice, we make use of the following fitted forms of the wave functions:

$$\Psi_0(\vec{k}) = a_1 e^{a_2|\vec{k}|^2 + a_3|\vec{k}| + a_4}$$



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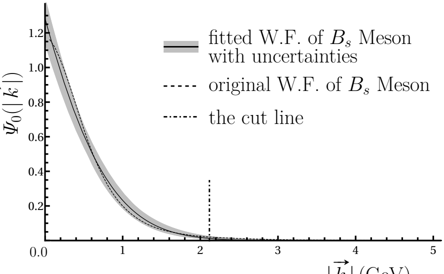

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Expressions of Decay Constants

In general, the decay constant of a pseudoscalar meson is defined by the matrix element of the axial current between the meson and the vacuum state:

$$\langle 0|\bar{q}\gamma^\mu\gamma^5 Q|P\rangle = i f_P P^\mu$$

The pseudoscalar meson as a bound state of a quark and antiquark system can be described by

$$\left|P(\vec{P})\right\rangle = \frac{1}{\sqrt{N_L}}\frac{1}{\sqrt{3}}\sum_i \int \mathrm{d}^3k_q\,\mathrm{d}^3k_Q\,\delta^{(3)}(\vec{P}-\vec{k}_q-\vec{k}_Q)\Psi_0(\vec{k}_q)$$
$$\frac{1}{\sqrt{2}}\left[c^{i\dagger}(\vec{k}_Q,\uparrow)b^{i\dagger}(\vec{k}_q,\downarrow)-c^{i\dagger}(\vec{k}_Q,\downarrow)b^{i\dagger}(\vec{k}_q,\uparrow)\right]\left|0\right\rangle$$

ACCMM Scenario

Holding the four-momentum conservation law, $k_q + k_Q = P$, we adopt the ACCMM scenario,

$$E_q + E_Q = m_P,$$
$$E_q^2 = m_q^2 + |\vec{k}|^2,$$
$$m_Q^2(\vec{k}) = E_Q^2 - |\vec{k}|^2.$$

and we also restrict the heavy quark mass, $m_Q(\vec{k}) \geq 0$, which responds to the cut lines in above figures.

The decay constant:

$$f_P = \sqrt{\frac{3}{(2\pi)^3 m_P}} \int \mathrm{d}^3k\, \Psi_0(\vec{k}) \frac{\left(E_q + m_q\right)\left(E_Q + m_Q\right) - |\vec{k}|^2}{\sqrt{E_q E_Q \left(E_q + m_q\right)\left(E_Q + m_Q\right)}},$$

Numerical Results

Parameters:

$$m_s = 0.32\,\mathrm{GeV}, \quad m_u = m_d = 0.06\,\mathrm{GeV}, \quad m_b = 4.99\,\mathrm{GeV},$$

and the mesons' masses are taken from PDG

$$m_B = 5.28\,\mathrm{GeV}, \quad m_{B_s} = 5.37\,\mathrm{GeV}.$$

Our prediction of B mesons' decay constants:

$$f_B = 219 \pm 15\,\mathrm{MeV}, \quad f_{B_s} = 266 \pm 19\,\mathrm{MeV}, \quad f_{B_s}/f_B = 1.21 \pm 0.09\,.$$

COMPARISON

Reference	Method	f_B (MeV)	f_{B_s} (Mev)	f_{B_s}/f_B
this work	RPM*	219 ± 15	266 ± 19	1.21 ± 0.09
Colangelo 91	RPM	230 ± 35	245 ± 37	1.07 ± 0.17
Cvetič 04	QM BS‡	196 ± 29	216 ± 32	1.10 ± 0.18
Badalian 07	FCM£	182 ± 8	216 ± 8	1.19 ± 0.03
Hwang 09	LFQM§	204 ± 31	270.0 ± 42.8	1.32 ± 0.08
HPQCD 11	LQCD (2+1)¶	–	$225{\pm}3{\pm}3$	–
FNAL/MILC 11	LQCD (2+1)	$196.9{\pm}5.5{\pm}7.0$	$242.0{\pm}5.1{\pm}8.0$	$1.229{\pm}0.013{\pm}0.02$
HPQCD 12	LQCD (2+1)	$191{\pm}1{\pm}8$	$228{\pm}3{\pm}10$	$1.188{\pm}0.012{\pm}0.01$
Narison 12	QCD SR†	206 ± 7	234 ± 5	1.14 ± 0.03
Gelhausen 13	QCD SR	207^{+17}_{-9}	242^{+17}_{-12}	$1.17^{+0.03}_{-0.04}$
HPQCD 13	LQCD (2+1+1)	184 ± 4	224 ± 5	1.217 ± 0.008
ETM 13	LQCD (2+1+1)	196 ± 9	235 ± 9	1.201 ± 25
Aoki 14	LQCD (2+1)	$218.8{\pm}6.4{\pm}30.8$	$263.5{\pm}4.8{\pm}36.7$	$1.193{\pm}0.020{\pm}0.04$
RBC/UKQCD 14	LQCD (2+1)	$195.6{\pm}6.4{\pm}13.3$	$235.4{\pm}5.2{\pm}11.1$	$1.223{\pm}0.014{\pm}0.07$
Wang 15	QCD SR	194 ± 15	231 ± 16	1.19 ± 0.10

* **Relativistic potential model.** † **QCD sum rules.** ‡ **Quark model based on Bethe-Salpeter equation.**

¶ **lattice-QCD with dynamical quark flavors N_f.** § **Light-front quark model.** £ **Field correlator method.**

BR of Leptonic Decays

The branching ratios of purely leptonic decays of B mesons,

$$\mathscr{B}\left(B^{\pm}\to l^{\pm}\nu\right)=\frac{G_F^2 m_l^2 m_B}{8\pi}\left(1-\frac{m_l^2}{m_B^2}\right)^2 f_B^2\left|V_{ub}\right|^2\tau_B,$$

In this work, we obtain

$$\mathscr{B}\left(B^{+}\to e^{+}\nu_e\right)=(1.17\pm 0.18)\times 10^{-11},$$
$$\mathscr{B}\left(B^{+}\to \mu^{+}\nu_\mu\right)=(5.01\pm 0.78)\times 10^{-7},$$
$$\mathscr{B}\left(B^{+}\to \tau^{+}\nu_\tau\right)=(1.41\pm 0.22)\times 10^{-4},$$

with CKM parameter form PDG,

$$|V_{ub}|=(4.09\pm 0.39)\times 10^{-3}.$$

Exp. Data

For instance,

Experiment	Tagging	$\mathscr{B}$(units of 10^{-4})
Belle	Hadronic	$0.72^{+0.27}_{-0.25} \pm 0.11$
Belle	Semileptonic	$1.25 \pm 0.28 \pm 0.27$
BABAR	Hadronic	$1.83^{+0.53}_{-0.49} \pm 0.24$
BABAR	Semileptonic	$1.7 \pm 0.8 \pm 0.2$

Table: Experimental results for $\mathscr{B}(B^+ \to \tau^+ \nu_\tau)$.

D.A. in Coordinate Space (1)

Generalizing the current in the definition of the decay constant to nonlocal operators and utilizing Fierz identity, we obtain the matrix element:

$$\begin{aligned}
&\tilde{\Phi}_{\alpha\beta}(z) \equiv \left\langle 0 \middle| \bar{q}_\beta(z)[z,0]Q_\alpha(0) \middle| \bar{B}(P) \right\rangle \\
= &\frac{1}{4}\left\langle 0 \middle| \bar{q}(z)Q(0) \middle| \bar{B} \right\rangle I_{\alpha\beta} + \frac{1}{4}\left\langle 0 \middle| \bar{q}(z)\gamma^5 Q(0) \middle| \bar{B} \right\rangle (\gamma^5)_{\alpha\beta} \\
&+\frac{1}{8}\left\langle 0 \middle| \bar{q}(z)\sigma^{\mu\nu}\gamma^5 Q(0) \middle| \bar{B} \right\rangle (\sigma_{\mu\nu}\gamma^5)_{\alpha\beta} + \frac{1}{4}\left\langle 0 \middle| \bar{q}(z)\gamma^\mu Q(0) \middle| \bar{B} \right\rangle (\gamma_\mu)_{\alpha\beta} \\
&-\frac{1}{4}\left\langle 0 \middle| \bar{q}(z)\gamma^\mu\gamma^5 Q(0) \middle| \bar{B} \right\rangle (\gamma_\mu\gamma^5)_{\alpha\beta},
\end{aligned}$$

where $[z, 0]$ **stands for the path-ordered exponential, which is called Wilson line that connects the point** 0 **and** z**:**

$$[z,0] \equiv \mathrm{Pexp}\left(i\int_0^z \mathrm{d}x^\mu A_\mu(x) \right).$$

D.A. in Coordinate Space (2)

Using the similar calculations as in decay constants, we obtain:

$$
\begin{aligned}
\left\langle 0\left|\bar{q}(z)Q(0)\right|\bar{B}\right\rangle &= 0,\\
\left\langle 0\left|\bar{q}(z)\gamma^5 Q(0)\right|\bar{B}\right\rangle &= -if_B m_B\tilde{\phi}_P,\\
\left\langle 0\left|\bar{q}(z)\sigma^{\mu\nu}\gamma^5 Q(0)\right|\bar{B}\right\rangle &= -if_B\tilde{\phi}_T\left(P^\mu z^\nu - P^\nu z^\mu\right),\\
\left\langle 0\left|\bar{q}(z)\gamma^\mu Q(0)\right|\bar{B}\right\rangle &= 0,\\
\left\langle 0\left|\bar{q}(z)\gamma^\mu\gamma^5 Q(0)\right|\bar{B}\right\rangle &= f_B\left(i\tilde{\phi}_{A1}P^\mu - m_B\tilde{\phi}_{A2}z^\mu\right).
\end{aligned}
$$

Introducing a few conventions, $N_B \equiv \frac{i}{f_B}\sqrt{\frac{3}{(2\pi)^3 m_B}}$**,** $k_q = (E_q, \vec{k})$**, and**

$$
A_T(k^1,k^2,k^3) \equiv \Psi_0(\vec{k})\frac{E_Q+m_Q+E_q+m_q}{\sqrt{E_qE_Q(E_q+m_q)(E_Q+m_Q)}},\quad A_+(k^1,k^2,k^3) \equiv \Psi_0(\vec{k})\frac{(E_q+m_q)(E_Q+m_Q)+|\vec{k}|^2}{\sqrt{E_qE_Q(E_q+m_q)(E_Q+m_Q)}}
$$

$$
A(k^1,k^2,k^3) \equiv \Psi_0(\vec{k})\frac{E_Q+m_Q-E_q-m_q}{\sqrt{E_qE_Q(E_q+m_q)(E_Q+m_Q)}},\quad A_-(k^1,k^2,k^3) \equiv \Psi_0(\vec{k})\frac{(E_q+m_q)(E_Q+m_Q)-|\vec{k}|^2}{\sqrt{E_qE_Q(E_q+m_q)(E_Q+m_Q)}},
$$

D.A. in Coordinate Space (3)

where the four independent D.A.s are

$$\tilde{\phi}_P(z) = N_B \int \mathrm{d}^3k \left[-A_+(k^1,k^2,k^3)\right] e^{-ik_q\cdot z},$$

$$\tilde{\phi}_T(z) = N_B \int \mathrm{d}^3k \left[\frac{1}{3}\sum_i \int_0^{k^i} A_T(\eta,\ldots)\eta\,\mathrm{d}\eta\right] e^{-ik_q\cdot z},$$

$$\tilde{\phi}_{A2}(z) = N_B \int \mathrm{d}^3k \left[\frac{1}{3}\sum_i \int_0^{k^i} A(\eta,\ldots)\eta\,\mathrm{d}\eta\right] e^{-ik_q\cdot z},$$

$$\tilde{\phi}_{A1}(z) = N_B \int \mathrm{d}^3k \left[-A_-(k^1,k^2,k^3) - E_q A(k^1,k^2,k^3)\right] e^{-ik_q\cdot z}.$$

for the expressions with ellipsis in parenthesis:

$$\sum_i \int_0^{k^i} A_T(\eta,\ldots)\eta\,\mathrm{d}\eta = \int_0^{k^1} A_T(\eta,k^2,k^3)\eta\,\mathrm{d}\eta + \int_0^{k^2} A_T(k^1,\eta,k^3)\eta\,\mathrm{d}\eta + \int_0^{k^3} A_T(k^1,k^2,\eta)\eta\,\mathrm{d}\eta\,.$$

D.A. in Coordinate Space (4)

Combine these independent D.A.s together, we finally get the D.A. in coordinate space,

$$\tilde{\Phi}_{\alpha\beta}(z) = \frac{-if_B}{4}\left\{\left[m_B\tilde{\phi}_P + \frac{1}{2}\tilde{\phi}_T(P^\mu z^\nu - P^\nu z^\mu)\sigma_{\mu\nu} + \left(\tilde{\phi}_{A1}P^\mu + im_B\tilde{\phi}_{A2}z^\mu\right)\gamma_\mu\right]\gamma^5\right\}_{\alpha\beta}$$

Considering a decay process which can be expressed as a convolution from

$$F = \int \mathrm{d}^4 z\, \tilde{\Phi}_{\alpha\beta}(z)\tilde{T}_{\beta\alpha}(z).$$

Insert proper Fourier transformations,

$$F = \int \mathrm{d}^3 k\, \Phi_{\alpha\beta}(k)\, T_{\beta\alpha}(k)\Big|_{k^2=m_q^2}.$$

D.A. in Momentum Space (1)

Thus, the distribution amplitudes in momentum space are:

$$\Phi_{\alpha\beta}(k) = \left\{ \frac{-i f_B m_B}{4} \left[\phi_P(k) + \frac{i}{2}\phi_T(k)\sigma_{\mu\nu}\left(v^\mu \frac{\partial}{\partial k_\nu} - v^\nu \frac{\partial}{\partial k_\mu} \right) + \left(\phi_{A1}(k)\not{v} - \phi_{A2}(k)\gamma_\mu \frac{\partial}{\partial k_\mu} \right) \right] \gamma^5 \right\}_{\alpha\beta}$$

where,

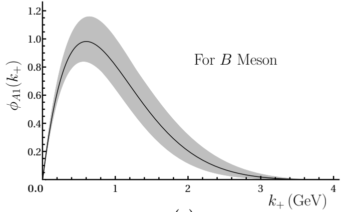
$$\phi_P(k_q^\mu) = -N_B A_+(k^1, k^2, k^3), \qquad \phi_T(k_q^\mu) = \frac{N_B}{3} \sum_i \int_0^{k^i} A_T(\eta, \dots)\eta \,\mathrm{d}\eta$$

$$\phi_{A1}(k_q^\mu) = -N_B \left[A_-(k^1, k^2, k^3) + E_q A(k^1, k^2, k^3) \right], \quad \phi_{A2}(k_q^\mu) = \frac{N_B}{3} \sum_i \int_0^{k^i} A(\eta, \dots)\eta \,\mathrm{d}\eta \,.$$

For comparison with generally used LCDA ϕ_B^+,

$$k_\pm = \frac{E_q \pm k^3}{\sqrt{2}}, \quad k_\perp^\mu = (0, k^1, k^2, 0), \quad \phi_B^+(x) \sim \phi_{A1}(k_+) \ \ \text{< heavy quark limit >}$$

D.A. in Momentum Space (2)



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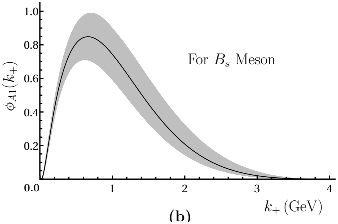

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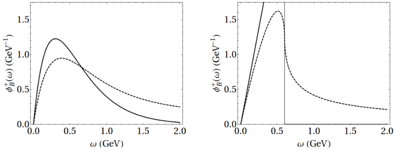
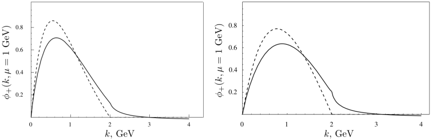




D.A. in Momentum Space (3)

For simplicity, we introduce a $K(\vec{k})$ function,

$$K(\vec{k}) \equiv \frac{-2N_B\Psi_0(\vec{k})}{\sqrt{E_q E_Q (E_q + m_q)(E_Q + m_Q)}}$$

then, combining all the independent D.A.s in momentum space together, we obtain a very concise form,

$$\Phi_{\alpha\beta}(k_q^\mu) = \frac{-if_B m_B}{4} K(\vec{k}) \left\{ \begin{pmatrix} (E_Q + m_Q) I_{2\times2} \\ \vec{k}\cdot\vec{\sigma} \end{pmatrix} \begin{pmatrix} \vec{k}\cdot\vec{\sigma} & (E_q + m_q) I_{2\times2} \end{pmatrix} \right\}_{\alpha\beta}$$

D.A. in Momentum Space (4)

For the convenience of connecting with other approaches, we re-express the above distribution amplitudes,

$$= \frac{-i f_B m_B}{4} K(\vec{k})$$

$$\cdot \left\{ (E_Q + m_Q) \frac{1 + \not{v}}{2} \left[\left(\frac{k_+}{\sqrt{2}} + \frac{m_q}{2} \right) \not{n}_+ + \left(\frac{k_-}{\sqrt{2}} + \frac{m_q}{2} \right) \not{n}_- - k_\perp^\mu \gamma_\mu \right] \gamma^5 \right.$$

$$\left. -(E_q + m_q) \frac{1 - \not{v}}{2} \left[\left(\frac{k_+}{\sqrt{2}} - \frac{m_q}{2} \right) \not{n}_+ + \left(\frac{k_-}{\sqrt{2}} - \frac{m_q}{2} \right) \not{n}_- - k_\perp^\mu \gamma_\mu \right] \gamma^5 \right\}_{\alpha\beta}.$$

Factorization Approach

In general, for a physics process, its matrix element F^μ, in factorization approach, can be illustrated as:

$$F^\mu = \Phi \otimes T$$

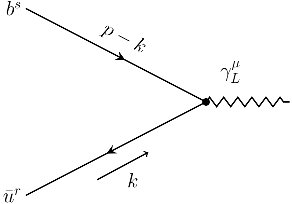
Also, we can expand them by the power of α_s in a perturbative way,

$$\begin{aligned} F^\mu &= F^{(0)\mu} + F^{(1)\mu} + \cdots = \Phi \otimes T \\ &= \left[\Phi^{(0)} \otimes T^{(0)}\right] + \left[\Phi^{(0)} \otimes T^{(1)} + \Phi^{(1)} \otimes T^{(0)}\right] + \cdots \end{aligned}$$

Therefor, up to one-loop level,

$$\Phi^{(0)} \otimes T^{(1)} = F^{(1)\mu} - \Phi^{(1)} \otimes T^{(0)}$$

TREE LEVEL



$$F_{b\bar{u}}^{(0)\mu} = \langle 0|\bar{u}\gamma_L^\mu b|\bar{u}^r(k)b^s(p-k)\rangle = \frac{1}{(2\pi)^3}\sqrt{\frac{m_u m_b}{k^0(p-k)^0}}\,\bar{v}^r(k)\gamma_L^\mu u^s(p-k)$$

$$\Phi_{\alpha\beta}^{(0)b\bar{u}}(\tilde{k}) = \int d^4z\, e^{i\tilde{k}\cdot z}\langle 0|\bar{u}_\beta(z)[z,0]b_\alpha(0)|\bar{u}^r(k)b^s(p-k)\rangle = \frac{1}{(2\pi)^3}\sqrt{\frac{m_u m_b}{k^0(p-k)^0}}(2\pi)^4\delta^{(4)}(\tilde{k}-k)\bar{v}^r_\beta(k)u^s_\alpha(p-k)$$

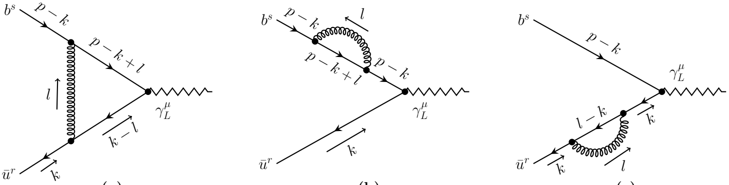
$$F_{b\bar{u}}^{(0)\mu} = \int\frac{d^4\tilde{k}}{(2\pi)^4}\Phi_{\alpha\beta}^{(0)b\bar{u}}(\tilde{k})T_{\beta\alpha}^{(0)}(\tilde{k}) = \frac{1}{(2\pi)^3}\sqrt{\frac{m_u m_b}{k^0(p-k)^0}}\,\bar{v}^r_\beta(k)T^{(0)}_{\beta\alpha}(k)u^s_\alpha(p-k)$$

And extract the hard-scattering kernel at tree level,

$$T_{\beta\alpha}^{(0)}(k) = \left(\gamma_L^\mu\right)_{\beta\alpha}.$$

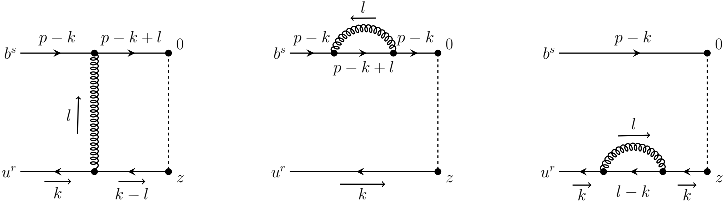
One-loop Level (1)

The one-loop Feynman diagrams for matrix element F^{μ},



(a) (b) (c)

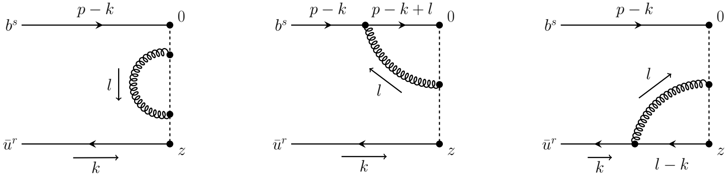
For distribution amplitude $\Phi_{\alpha\beta}$, part one,



(a) (b) (c)

One-loop Level (2)

For distribution amplitude $\Phi_{\alpha\beta}$, part two, related with gluon field in the Wilson line,



(a) (b) (c)

$$F_V^{(1)\mu} = \Phi_{\alpha\beta}^{(1)V} \otimes T_{\beta\alpha}^{(0)},$$
$$F_{bR}^{(1)\mu} = \Phi_{\alpha\beta}^{(1)b} \otimes T_{\beta\alpha}^{(0)},$$
$$F_{\bar{u}R}^{(1)\mu} = \Phi_{\alpha\beta}^{(1)\bar{u}} \otimes T_{\beta\alpha}^{(0)}.$$

Thus,

$$\Phi^{(0)} \otimes T^{(1)} = F^{(1)\mu} - \Phi^{(1)} \otimes T^{(0)} \implies T^{(1)} = 0.$$

Summary

- In the relativistic potential model, we obtain wave functions of B mesons via solving the Schrödinger-type wave equation.
- Based on the wave functions, we adopt ACCMM scenario to calculate decay constants of B mesons and branching ratios of the purely leptonic decays thereafter whose results are consistent with experiment data.
- Generalizing to non-local operator current, we further figure out the distribution amplitudes of B mesons including longitudinal and transverse momentum components which should be useful in the study of semileptonic and nonleptonic decays.
- In the end, as an application, we discuss the factorization of the purely leptonic decay up to one-loop level and find that the hard-scattering kernel contribution at one-loop is zero.

Outlook

1. **Making use of the distribution amplitudes, we shall continue to study the more complicated semileptonic and non-leptonic decay channels.**
2. **Carry forward the distribution amplitudes of B mesons to one-loop level and figure out the dominance of next-to-leading order contributions.**
3. **Try to bring the charm mesons into this research scenarios...**

THANK YOU

Thank You!